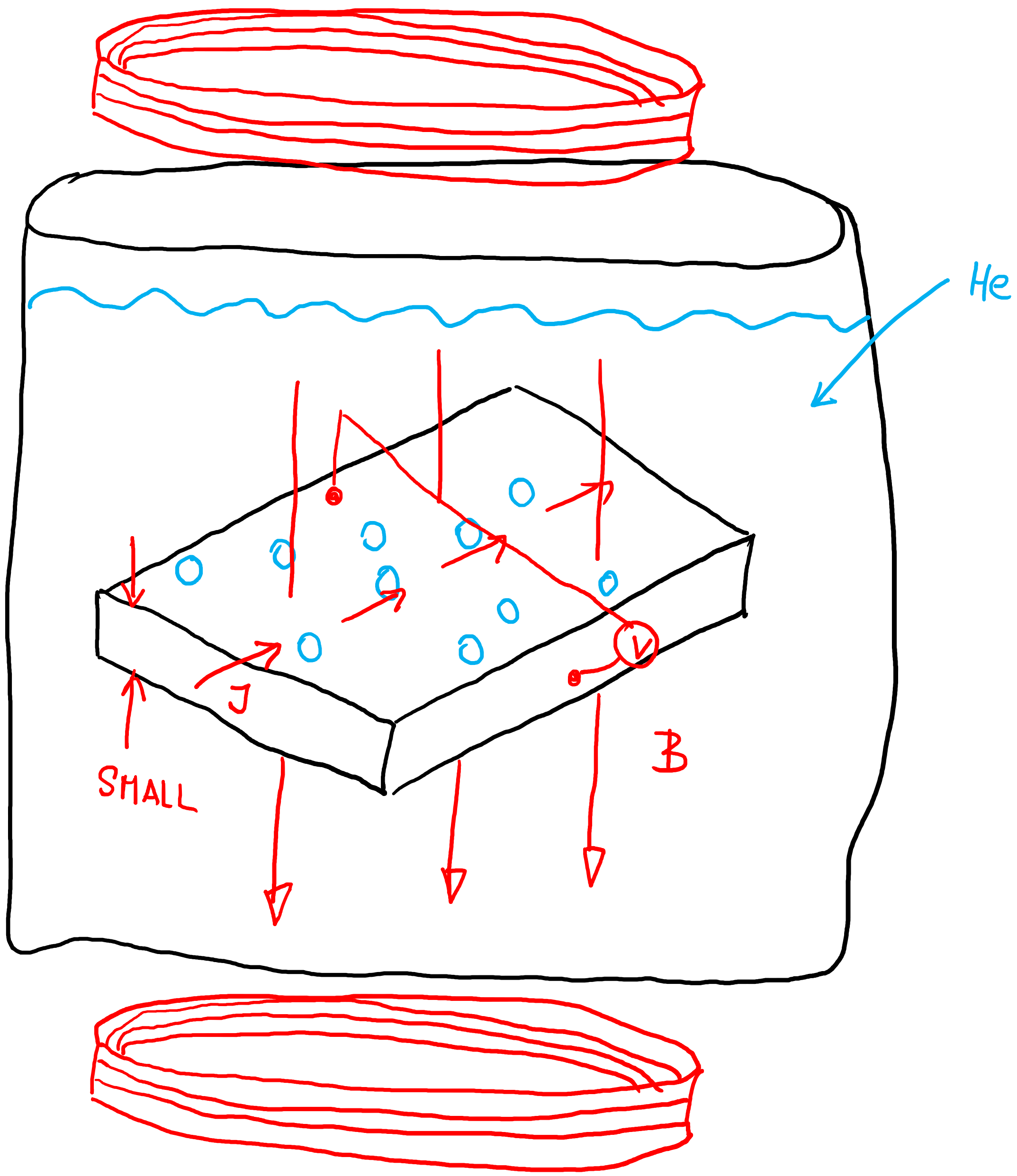
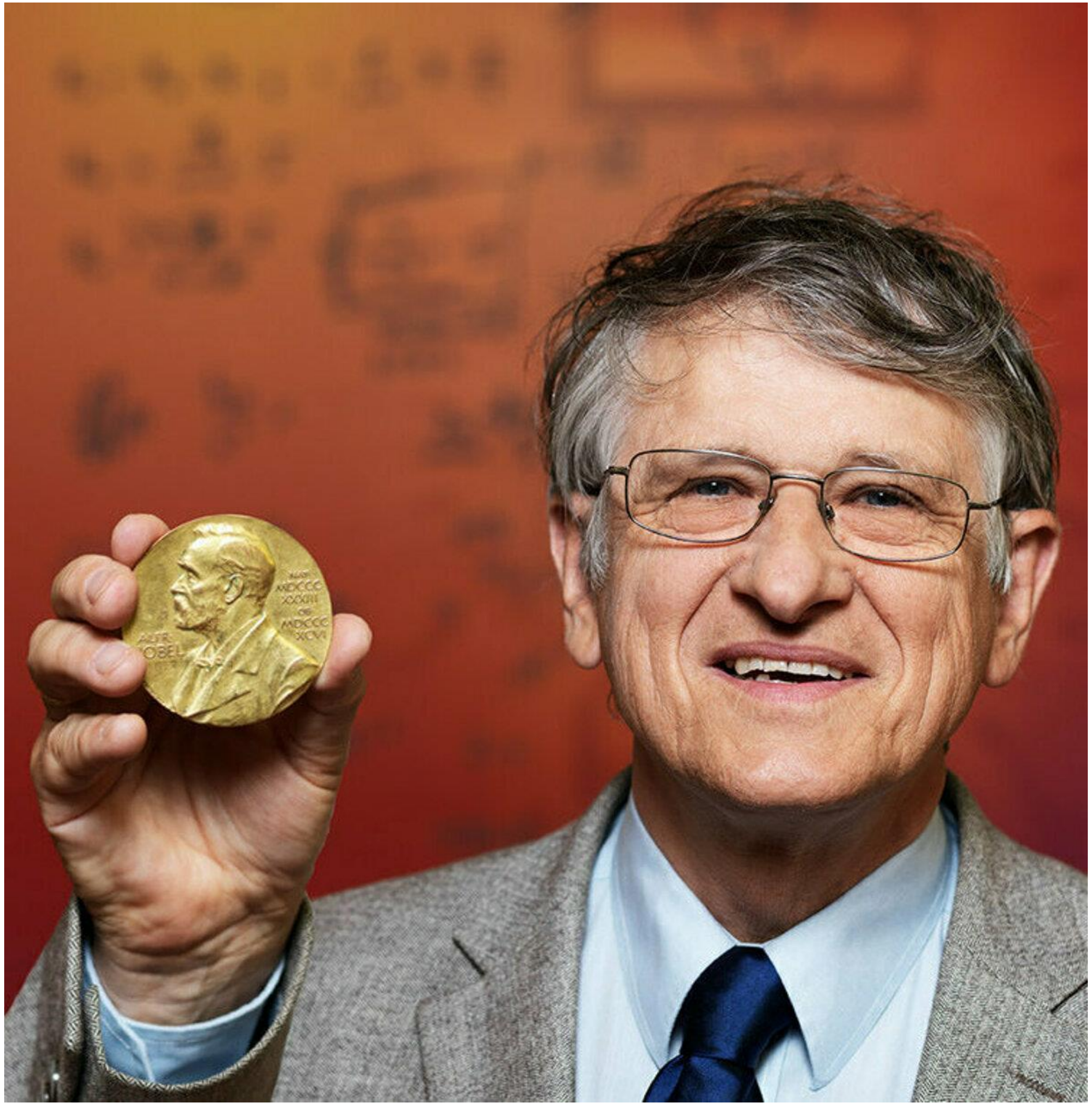
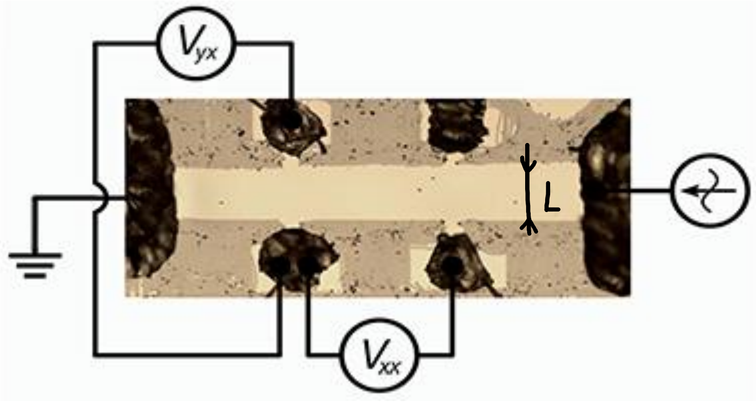
RECIPE ON GETTING FULLY QUANTUM:



RECIPE ON GETTING FULLY QUANTUM:

1) TAKE SOME ELECTRONS
2) LIMIT THEIR MOVEMENT IN 1 DIRECTION
3) COOL IT DOWN WELL
4) APPLY LARGE MAGNETIC FIELD
5) APPLY CURRENT, MEASURE VOLTAGE
6) ENJOY TOUCHING QUANTUM WORLD





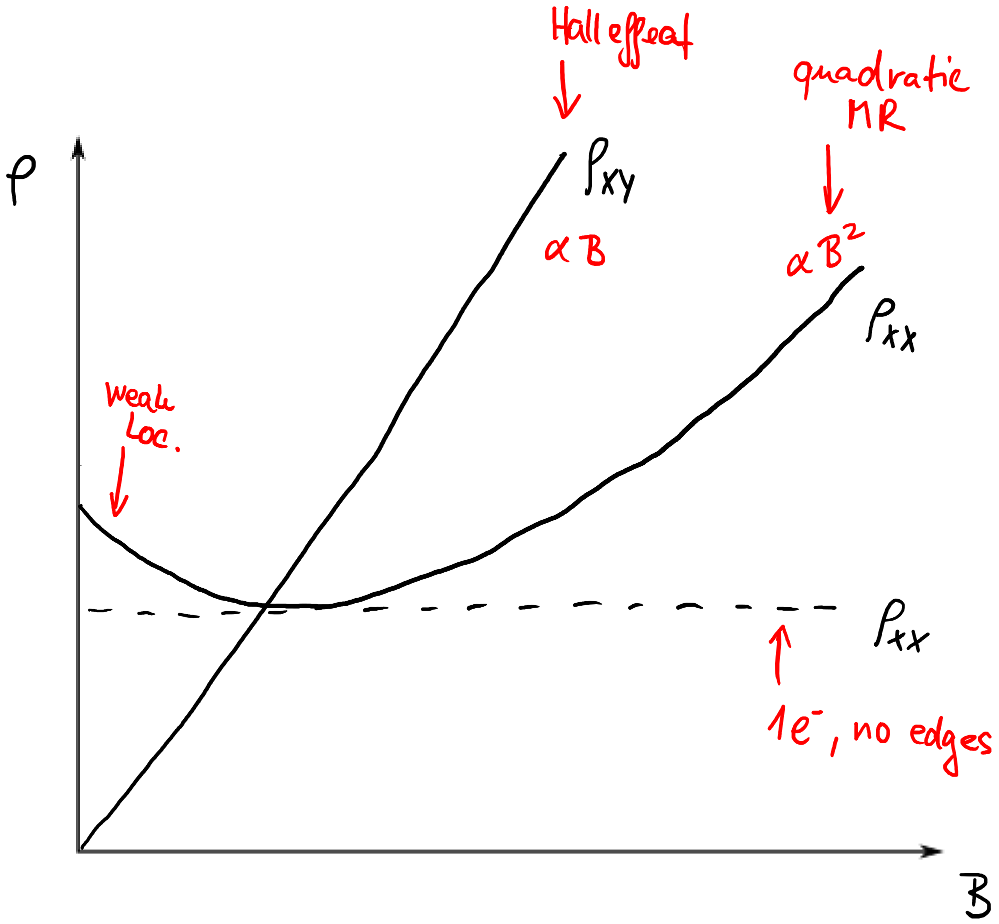
$$\underbrace{R_{xy}} = \frac{V_{xy}}{I_x} = \frac{E_{xy}}{L}\frac{L}{j_x} = \frac{E_{xy}}{j_x} = \underbrace{\rho_{xy}}$$

$$\bar{\rho} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{xy} & \rho_{xx} \end{pmatrix} \qquad \rho_{xy} = \frac{B_z}{ne} \qquad \rho_{xx} = \frac{m}{ne^2\tau}$$

ef. mass

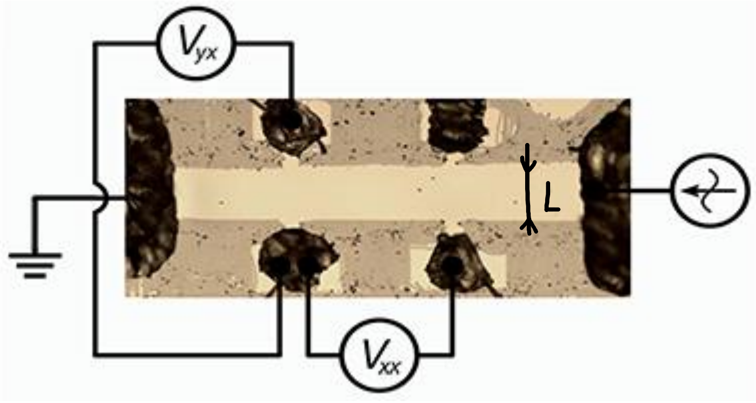
scatt. time

density



Bestwick, PRL (2015)

QUANTUM HALL EFFECT: INTRO



I

$$\underbrace{R_{xy}} = \frac{V_{xy}}{I_x} = \frac{E_{xy}}{L}\frac{L}{j_x} = \frac{E_{xy}}{j_x} = \underbrace{\rho_{xy}}$$

$$\bar{\rho} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{xy} & \rho_{xx} \end{pmatrix} \qquad \rho_{xy} = \frac{B_z}{ne} \qquad \rho_{xx} = \frac{m}{ne^2\tau}$$

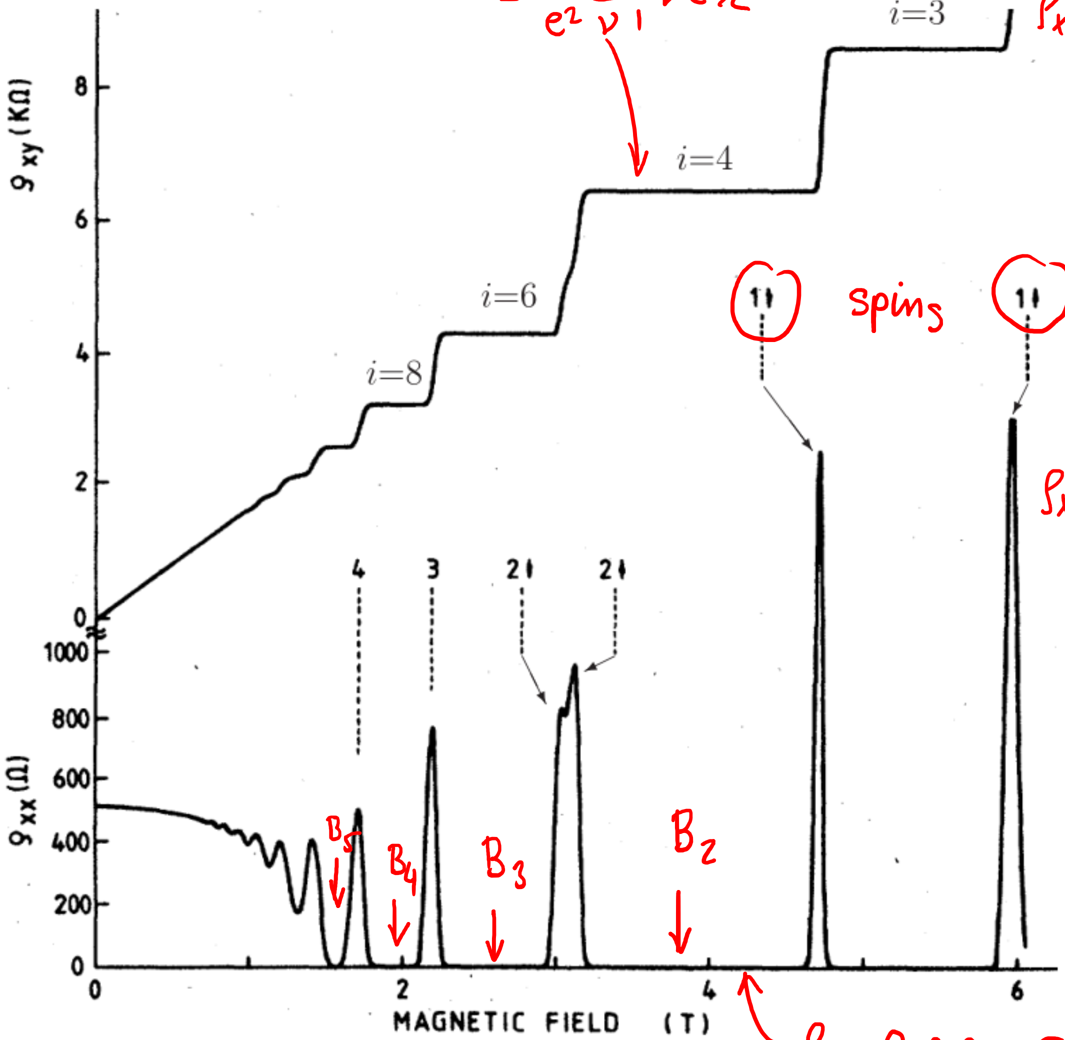
ef. mass

scatt. time

density

$\frac{h}{e^2} \sim 25{,}812\ k\Omega$ precision to 10 orders

$= \frac{h}{e^2}\frac{1}{\nu},\ \nu \in \mathbb{Z}$



ρ_{xy}

↑↓ spins ↑↓

ρ_{xx}

B_5 B_4 B_3 B_2

$\rho_{xx} = 0{,}000\ldots\ \Omega$

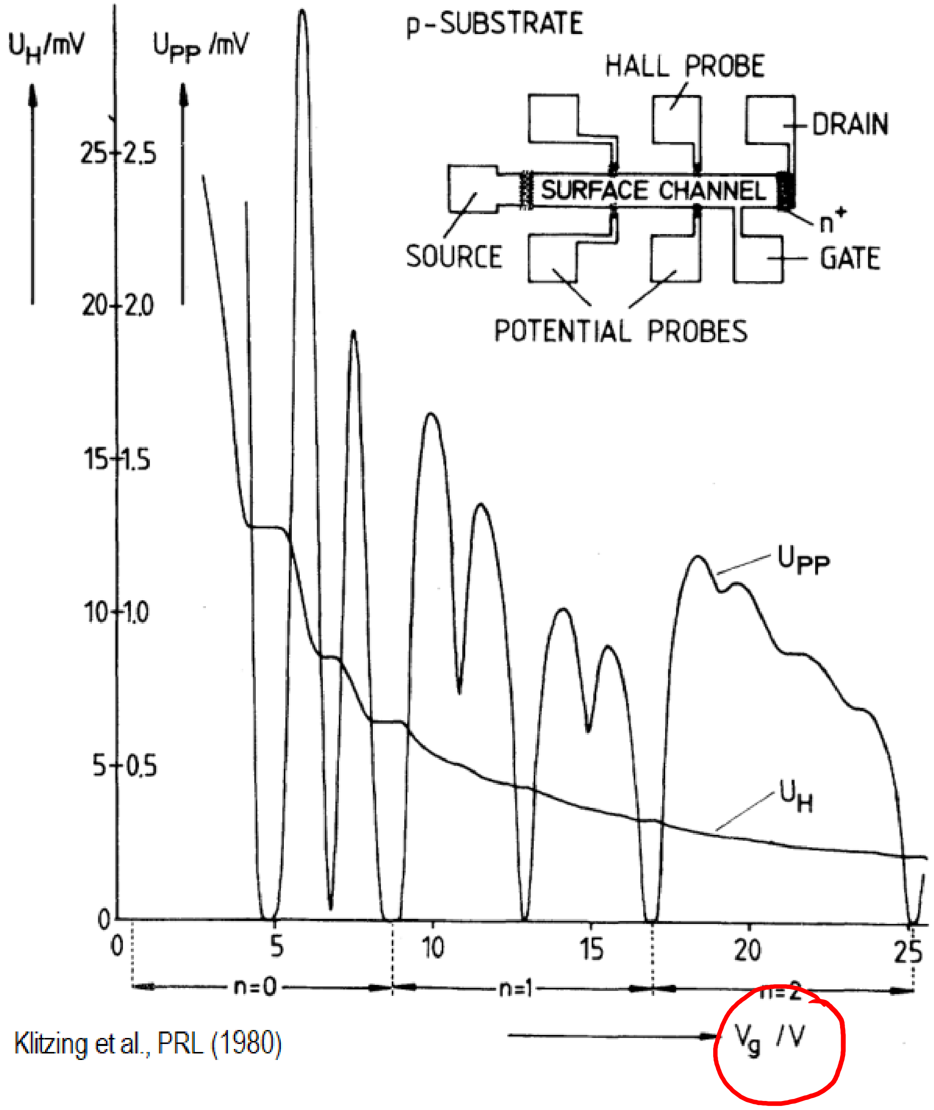
$< 10^{-10}\ \Omega/\square$

$\frac{B_2}{B_3} = \frac{3}{2}$

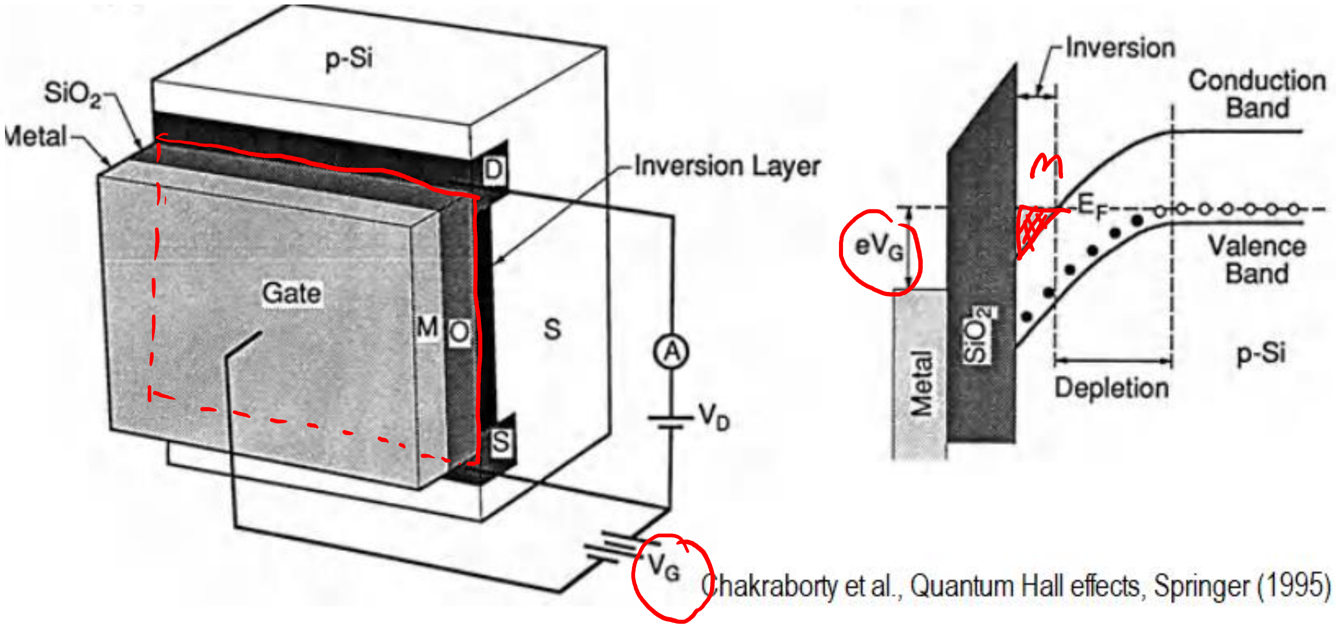
Bestwick, PRL (2015)

QUANTUM HALL EFFECT : INTRO

K Litzing obseved it vs V_g :

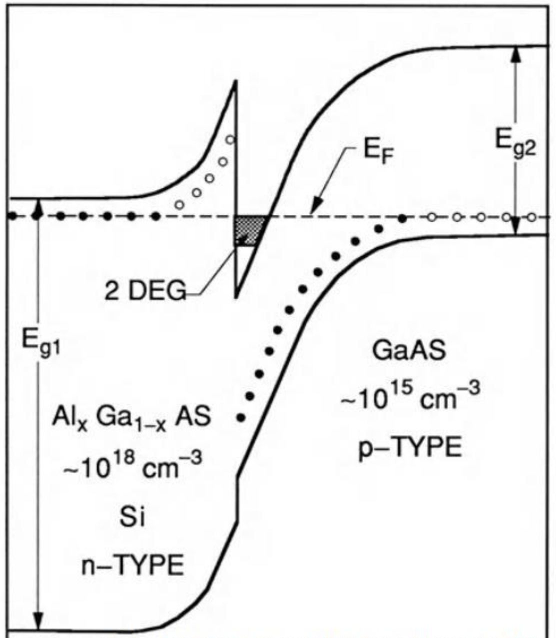


Klitzing et al., PRL (1980)



Chakraborty et al., Quantum Hall effects, Springer (1995)

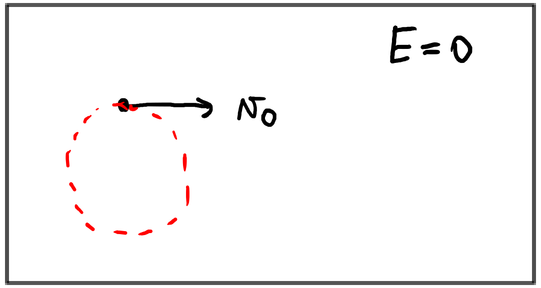
⇒ QHE will depend on carrier density

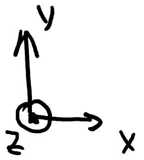


Chakraborty et al., Quantum Hall effects, Springer (1995)

QUANTUM HALL EFFECT : LANDAU LEVELS

CLASSICALLY :





$$m\dot{\vec{v}} = e(\vec{E} + \vec{v}\times\vec{B}) \qquad \vec{B} = (0,0,B)$$

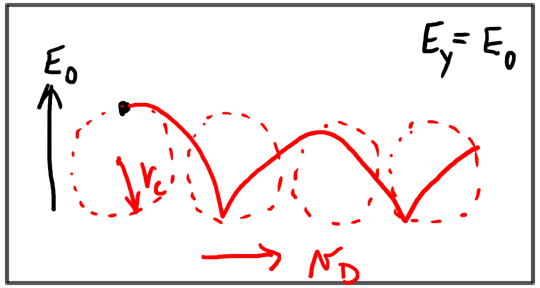
$$m\dot{v}_x = e v_y B$$
$$m\dot{v}_y = e v_x B \quad /\partial_t$$
$$m\dot{v}_z = 0$$

b.c. $v(0) = v_0$, $\dot{v}(0) = 0$

$$\frac{m}{eB}\ddot{v}_y = -e v_y \frac{B}{m}$$
$$\ddot{v}_y = -\omega_c^2 v_y \qquad \boxed{\omega_c = \frac{eB}{m}} \quad \text{CYCLOTRON FREQ}$$

Sol. $v_y(t) = C_1 e^{i\omega_c t} + C_2 e^{-i\omega_c t} \Rightarrow v_y(t) = v_0 \cos(\omega_c t)$, $v_x(t) = v_0 \sin(\omega_c t)$ $\Rightarrow$ $\left.\begin{matrix} x(t) \\ y(t)\end{matrix}\right\}$ $\dfrac{v_0}{\omega_c} \begin{matrix}\cos \\ \sin\end{matrix}\, \omega_c t$

$r_c = E_0/B$



$$v_x(t) = \underbrace{-\frac{E_0}{B}}_{v_D} + \frac{E_0}{B}\cos\omega_c t$$
$$v_y(t) = \frac{E_0}{B}\sin\omega_c t$$

it drifts $\perp$ to applied E

But ρ_{xy} contains h $\rightarrow$ we have to go quantum to explain it.

QUANTUM APPROACH:

$$\boxed{\hat{H} = \underbrace{\frac{1}{2m}\left[\hat{\vec{p}} + e\vec{A}\right]^2}_{\text{KINETIC}} + \underbrace{e\vec{E} + V}_{\text{POTENTIAL}} + \underbrace{g\mu_B B\,\hat{S}_z}_{\text{SPIN}}}$$

$\vec{E} = 0$ (now)

$\vec{B} = (0,0,B)$

vector potential

$\vec{A}$: $\nabla \times \vec{A} = \vec{B} \Rightarrow (\nabla \times \vec{A})_z = \frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x$

$\nabla \times = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times$

Landau gauge:

$\underline{A = (0, xB, 0)}$

$$\hat{H} = \frac{1}{2m}\hat{p}_x^2 + \frac{1}{2m}\left[\hat{p}_y + e\hat{x}B\right]^2$$ (no motion in $\hat{z} \rightarrow \hat{p}_z = 0$)

let's get rid of p_y

Two ways: 1. $[\hat{H}, \hat{p}_y] = 0$ since there is no $\hat{y}$ $\Rightarrow$ replace $\hat{p}_y$ by eigenvalue: $\hat{p}_y \rightarrow \hbar k_y$

or

2. Let's use $\psi_k(x,y) = e^{ik_y y} \cdot \chi_k(x)$

we feel it will be plain wave in $\hat{y}$

Apply $\hat{H}$ on ψ_k $\Rightarrow$ expand $[\;]^2$, apply $\hat{p}_y = i\hbar\frac{\partial}{\partial y} \cdot \rightarrow \hbar k_y$, again collaps to $[\;]^2$

$$\Rightarrow \hat{H} = \frac{1}{2m}\hat{p}_x^2 + \frac{1}{2m}\left[\hbar k_y + e\hat{x}B\right]^2 \overset{\text{rewrite}}{=} \boxed{\frac{1}{2m}\hat{p}_x^2 + \frac{m\omega_c^2}{2}\underbrace{\left[\hat{x} + k_y l_B^2\right]^2}_{(x-x_0)^2}}$$

hamiltonian of harm. oscillator

shift

$x_0 = -k_y l_B^2$

cyclotron fr.

$\omega_c = \frac{eB}{m}$

$l_B = \sqrt{\frac{\hbar}{eB}}$

magnetic length

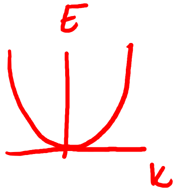
QUANTUM HALL EFFECT : LANDAU LEVELS

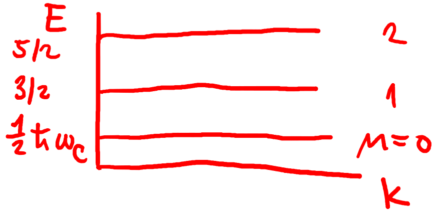
Quantum oscillator : $\hat{H} = \frac{1}{2m}p_x^2 + \frac{m\omega_c^2}{2}\left[x + k_y \ell_B^2\right]^2$

$\omega_c = \frac{eB}{m}$

$\ell_B = \sqrt{\frac{\hbar}{eB}}$

→ Eigen energy cannot depend on position of oscillator : $E_n = \hbar\omega_c(n + 1/2)$ ↔ Free particle $E_k = \frac{\hbar^2 k^2}{2m}$





Landau levels

LOCALIZED

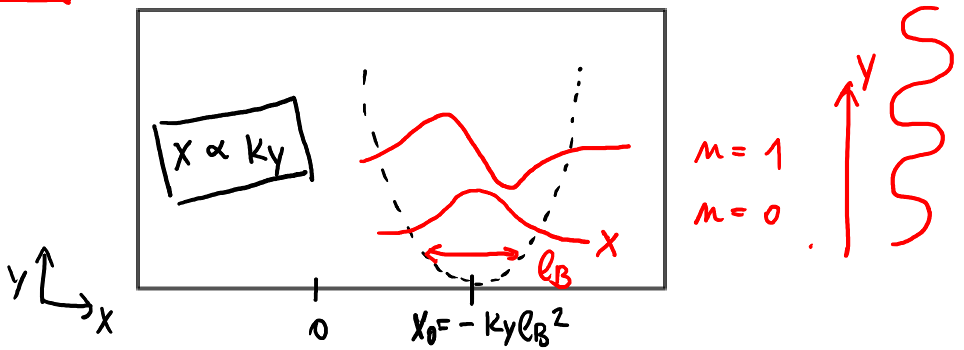
And motion ? "$v = \frac{1}{\hbar}\frac{\partial E}{\partial k} = 0$"

no group motion ! ⇔ classical picture

→ Eigen function : $\Psi_{k,n} \propto \underbrace{e^{ik_y y}}_{\text{PLAIN WAVE IN } y} \underbrace{\underbrace{H_n\left(\frac{x + k_y \ell_B^2}{\ell_B}\right)}_{\text{HERMIT. POL.}} \underbrace{e^{-(x + k_y \ell_B^2)^2 / 2\ell_B^2}}_{\text{GAUSSIAN}}}_{\text{Shifted}}$

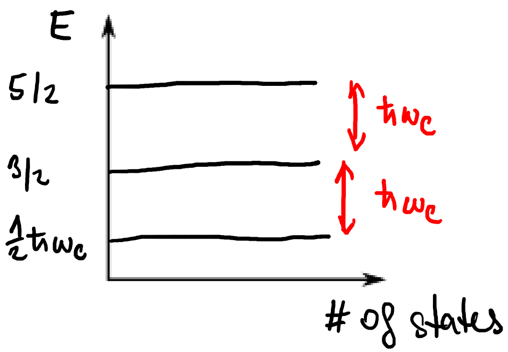
QUANTUM OSC :

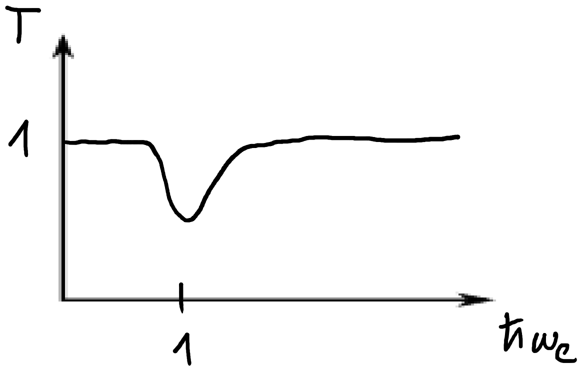
$$\psi_n(x) = e^{-\frac{m\omega x^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)$$



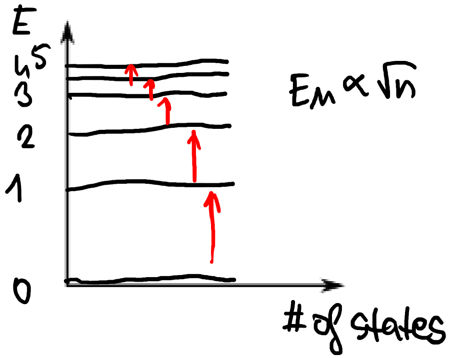
QUANTUM HALL EFFECT : LANDAU LEVELS

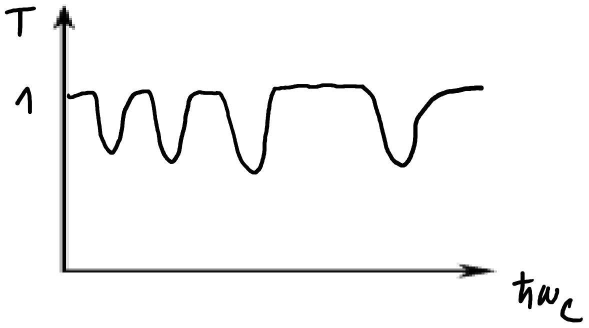
SPECTROSCOPY OF LANDAU LEVELS

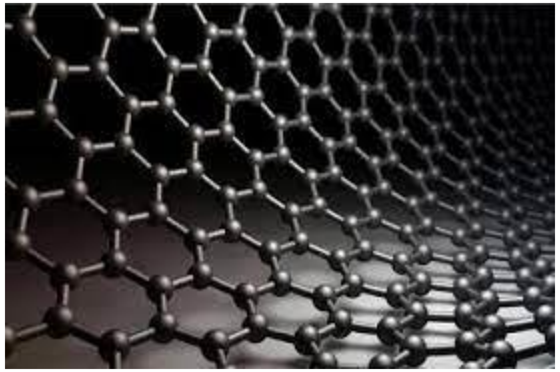


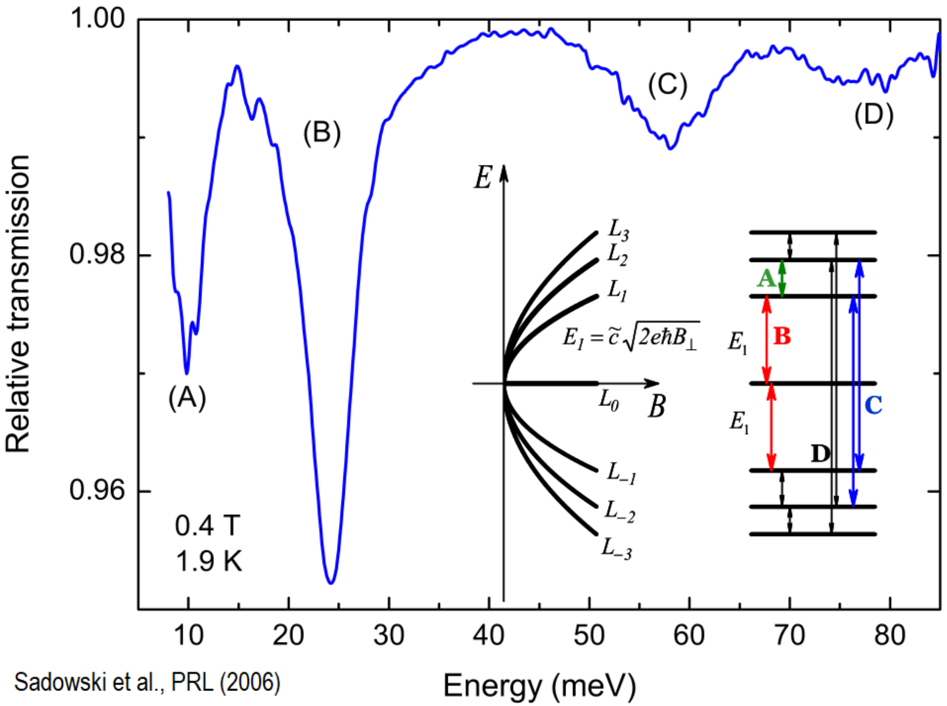


Normal system (GaAs QW)







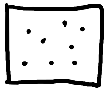


QUANTUM HALL EFFECT: DENSITY OF STATES

→ $\psi_{n,k}$ depends on n and k but E_n only on n ⇒ huge degeneration (all k have same energy) w/ 1 n

How huge is HUGE?

① sample



L_y

L_x

$x = -k_y \ell_B^2$

in $\hat{y}$: plain waves & particle in a box: $dk_y = \frac{2\pi}{L_y}$

# of states = # of boxes

what is k_y^{max}?

in $\hat{x}$: localized within ℓ_B: $0 < x < L_x$ $\quad x \to k_y$

$$\boxed{0 < k_y < \frac{L_x}{\ell_B^2}}$$

this is maximal k_y

⇒ $N = \frac{k_y^{max}}{dk_y}$ or better scientifically

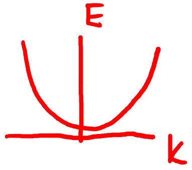
$$N = \int_0^{k_y^{max}} \frac{L_y}{2\pi}\, dk_y = \frac{L_y}{2\pi}\frac{L_x}{\ell_B^2}$$

$$n = \frac{N}{L_x L_y} = \frac{eB}{2\pi\hbar} = \boxed{\frac{eB}{h} = n_L = \frac{B}{\Phi_0}}$$

magnetic flux

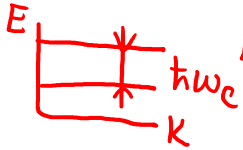
density of 1LL

② $B=0$ 2DEG



$dn = g(E)^{2D}\, dE$

$B \neq 0$



we have to get there same number of electrons

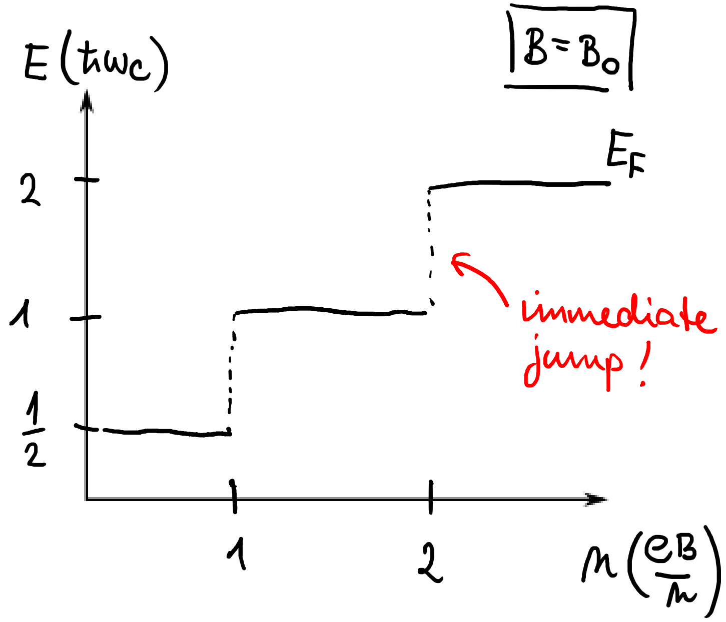
$$\boxed{dn} = \frac{g_s m}{2\pi\hbar^2}\underbrace{\hbar\omega_c}_{dE} = \frac{g_s m}{2\pi\hbar^2}\hbar\frac{eB}{m} = \boxed{g_s\frac{eB}{h}}$$

same

spin degeneration = 2

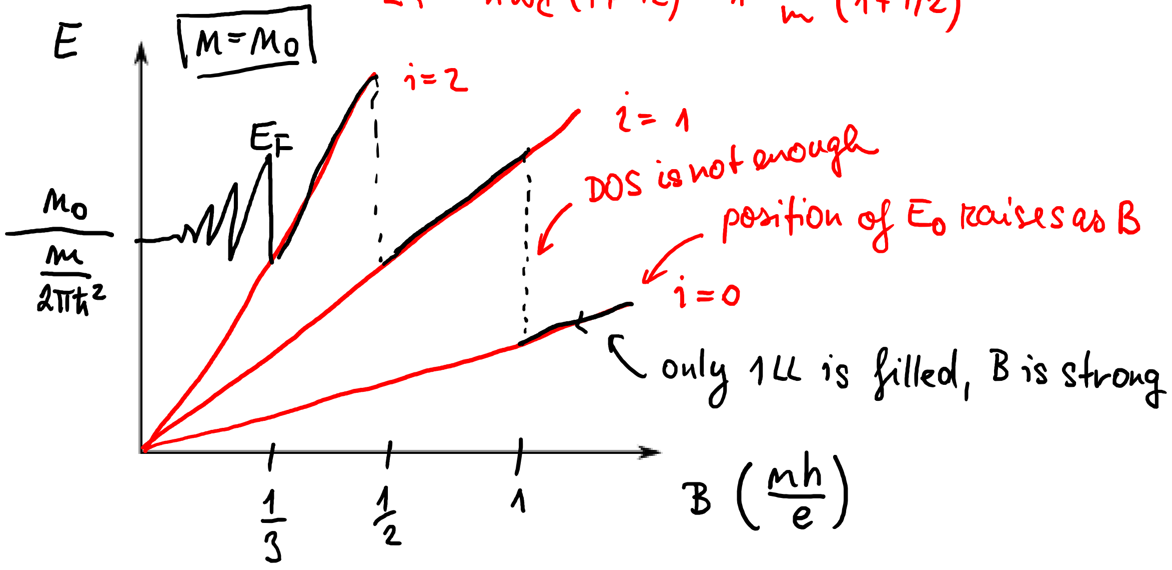
QUANTUM HALL EFFECT: DENSITY OF STATES

Let's now turn knobs! Where is Fermi level?



$n_{LL} = \frac{eB}{h}$ (# of states in 1 LL)

$E_i = \hbar\omega_c(i+1/2) = \hbar\frac{eB}{m}(i+1/2)$



Filling factor ν: $\boxed{n = \frac{eB}{h}\nu}$ $\nu \in \mathbb{Z}$

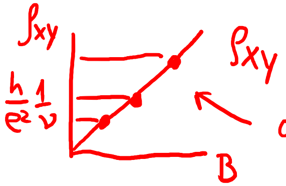
total density

# of states in 1 LL

remember $\longrightarrow$ $\rho_{xy} = R_{xy} = \frac{B}{ne} = \frac{h}{e^2}\frac{1}{\nu}$:)

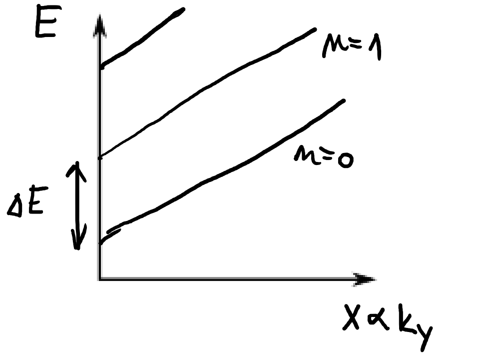
still linear in B, no steps

Plateaux in ρ_{xy} in QHE



already quantum numbers → we are close :)

QUANTUM HALL EFFECT: EDGE STATES



degeneration removed

Apply E_x:

$$\hat{H} = \frac{1}{2m}\hat{p}_x^2 + \frac{1}{2m}\left[\hat{p}_y + eBx\right]^2 + eEx$$

$$\downarrow$$

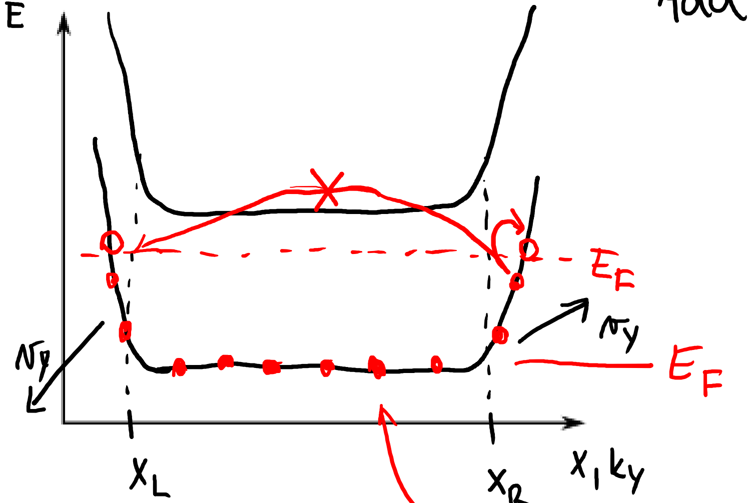
$$E_n = \underbrace{\hbar\omega_c\left(n+1/2\right)}_{\text{unperturbed LL}} + eE\left(\underbrace{k_y \ell_B^2}_{\propto X} - \frac{eE}{m\omega_c^2}\right) + \frac{m}{2}\frac{E^2}{B^2}$$

$$v_y = \frac{1}{\hbar}\frac{\partial E_n}{\partial k_y} = \frac{1}{\hbar} eE\ell_B^2 = \frac{E}{B} \qquad v_x = 0$$

no dep. on k_x or $\hat{y}$

motion in $\hat{y}$ (as in classical approach)

Add edges: $\hat{H} \to \hat{H} + V(x)$



Localized states

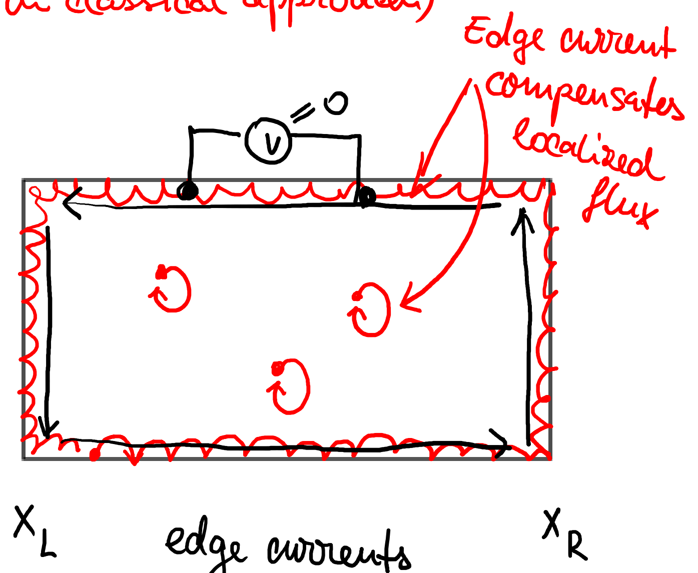
$$v_y = \frac{1}{\hbar}\frac{\partial E}{\partial k_y} = \frac{1}{\hbar}\frac{\partial V}{\partial k_y} \overset{k_y \leftrightarrow x}{=} \frac{1}{\hbar}\ell_B^2\frac{\partial V}{\partial x} = \frac{1}{eB}E(x)$$

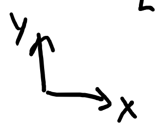
internal edge E

no over-bar scattering is allowed

→ no net current

→ no potential drop / resistance at edges

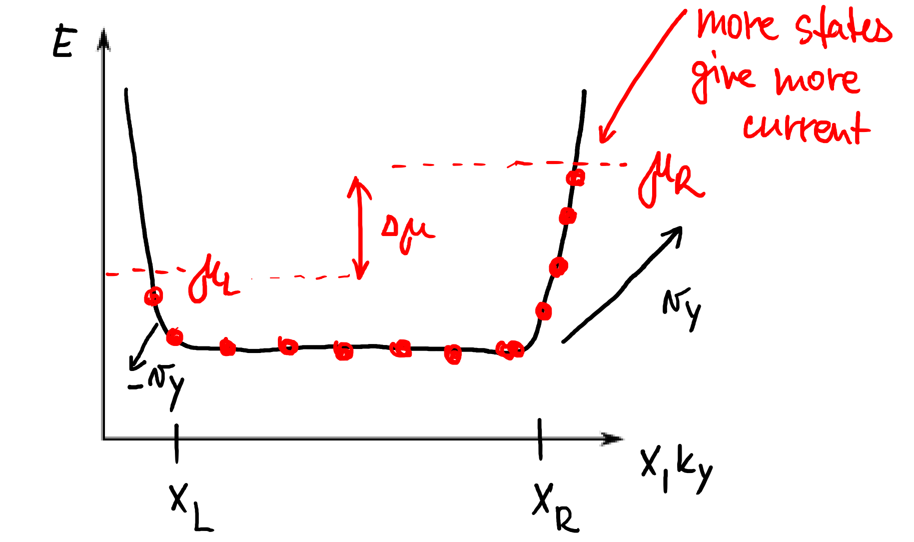


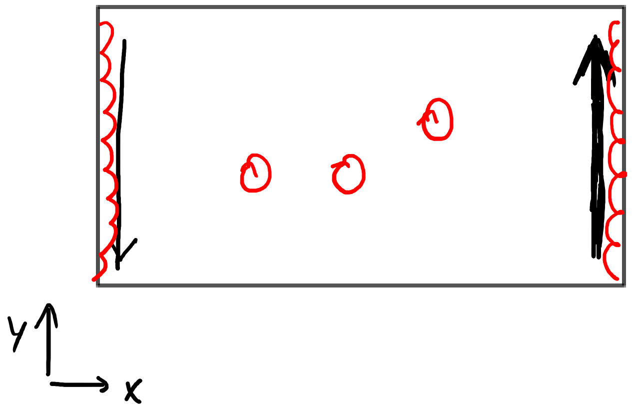


edge currents

$$\boxed{\rho_{xx} = 0}$$

QUANTUM HALL EFFECT: EDGE STATES



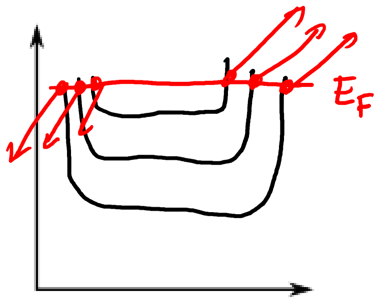


If we want a non-zero net current in $\hat{y}$?

$$\ell_B^2 = \frac{\hbar}{eB} \qquad \hbar = \frac{h}{2\pi}$$

$$I_y = -e\int \frac{dk_y}{2\pi}\, v_y(k_y) \overset{dk_y \to dx}{=} \frac{e}{2\pi \ell_B^2}\int d\cancel{x}\, \frac{1}{eB}\frac{\partial V}{\partial \cancel{x}} = \frac{e}{h}\int dV = \frac{e}{h}\Delta\mu$$

$$\boxed{\sigma_{xy} = \frac{I_y}{V_H} = \frac{I_y}{\frac{\Delta\mu}{e}} = \frac{e^2}{h}}$$



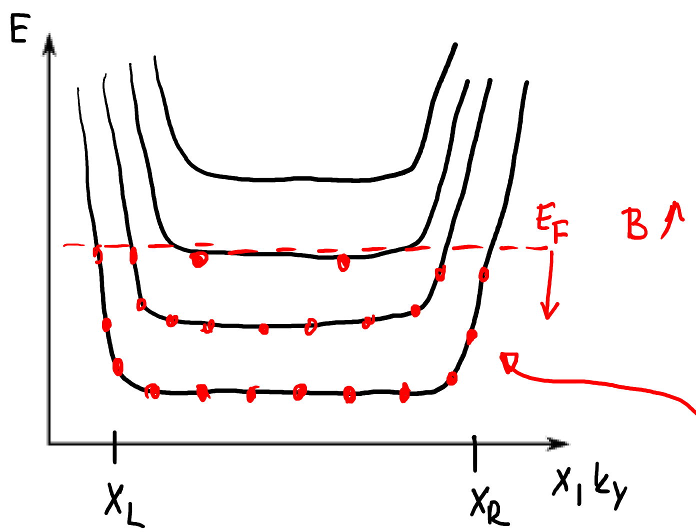
$$\Rightarrow \quad \sigma_{xy} = \frac{e^2}{h}\, i$$

$$\rho_{xy} = \frac{h}{e^2}\frac{1}{\nu} \quad \rightarrow \quad \frac{B}{he}$$

$$\rho_{xx} = 0 \quad \rightarrow \quad \neq 0$$

E_F is: Between LL (edge states) | inside a LL

QUANTUM HALL EFFECT : EDGE STATES



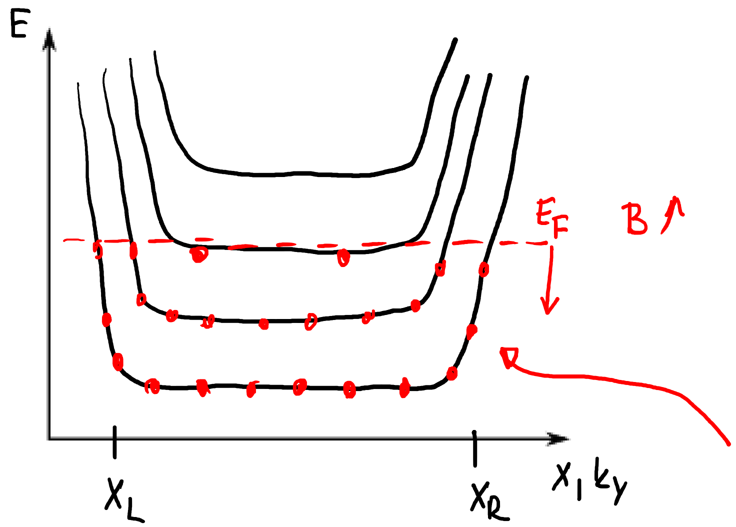
→ E_F MUST go "slowly" down, stay between LLs when B↑ to form edge states and quantum regime ($\rho_{xx}=0$, $\rho_{xy} = \frac{h}{e^2}\frac{1}{\nu}$)

→ if there are no states between LLs, E_F jumps abruptly from one LL to another ⇒ no quantum regime

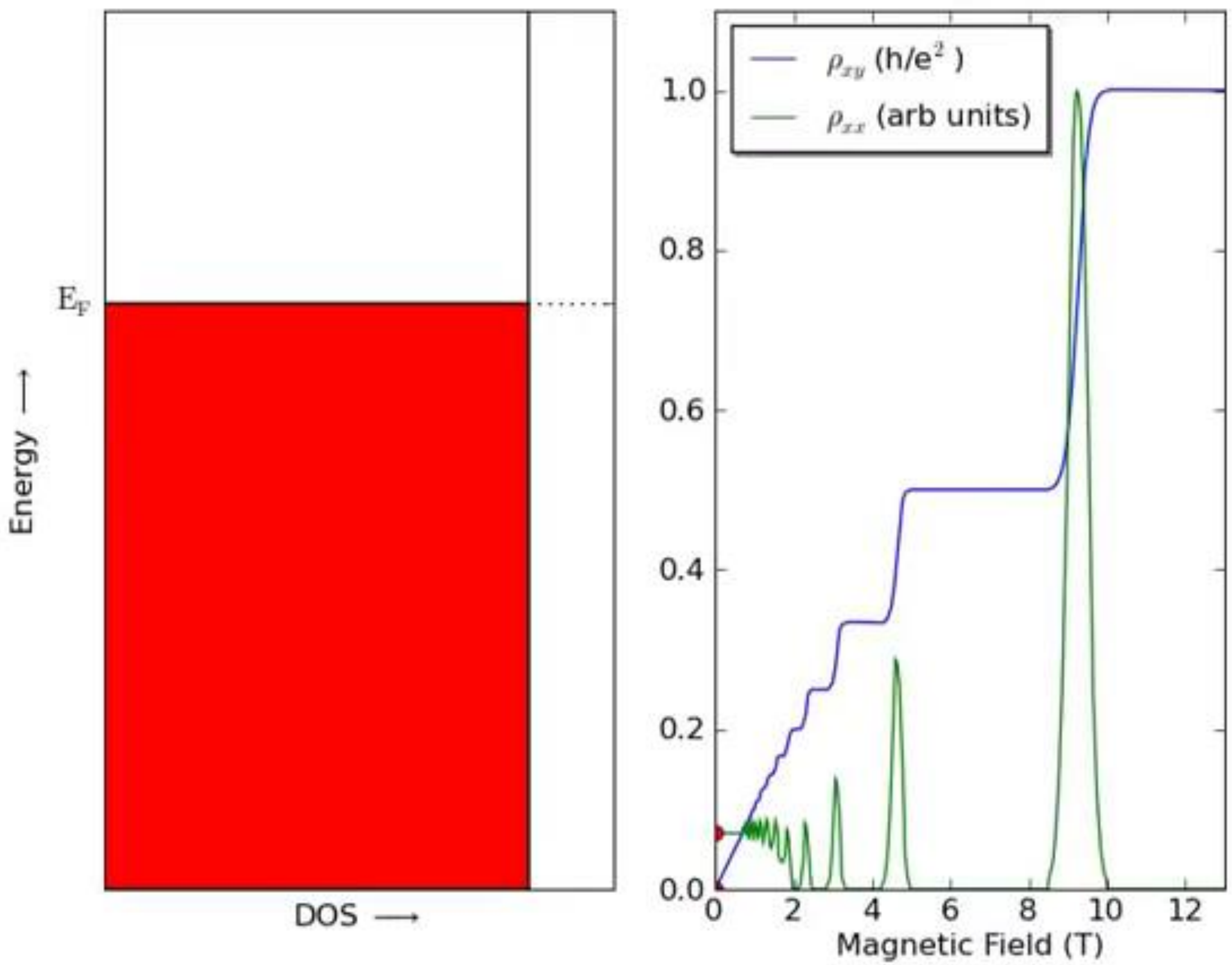
DOS of edge states is tiny → it doesn't help

⇒ BROADENING OF LLs !

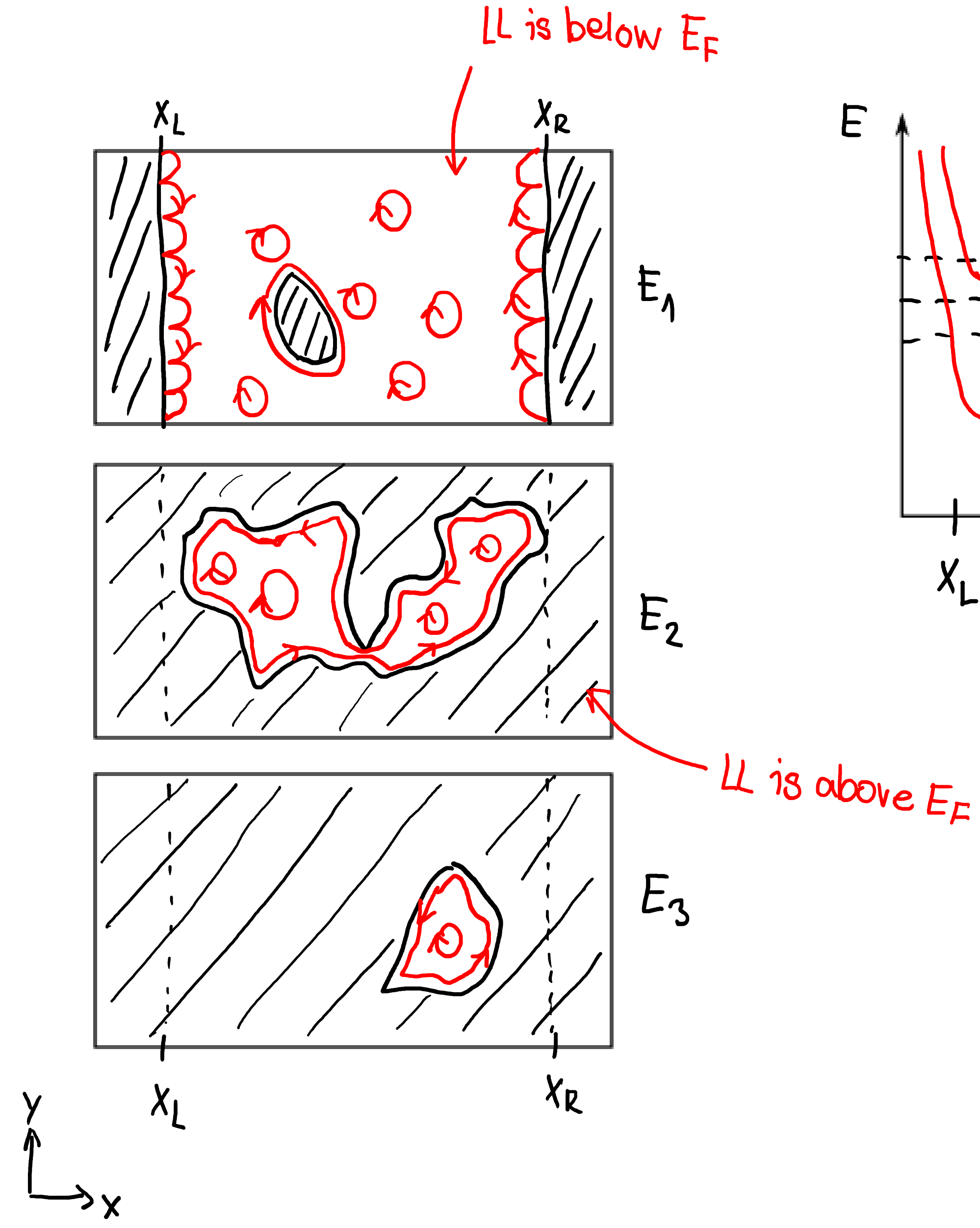
QUANTUM HALL EFFECT: EDGE STATES

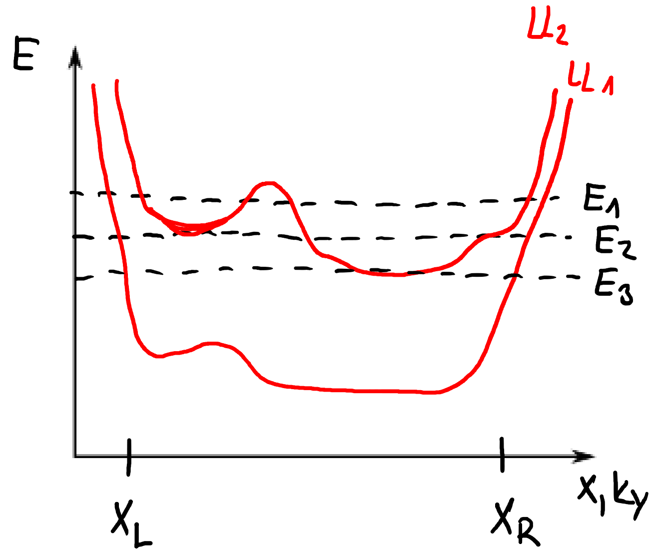


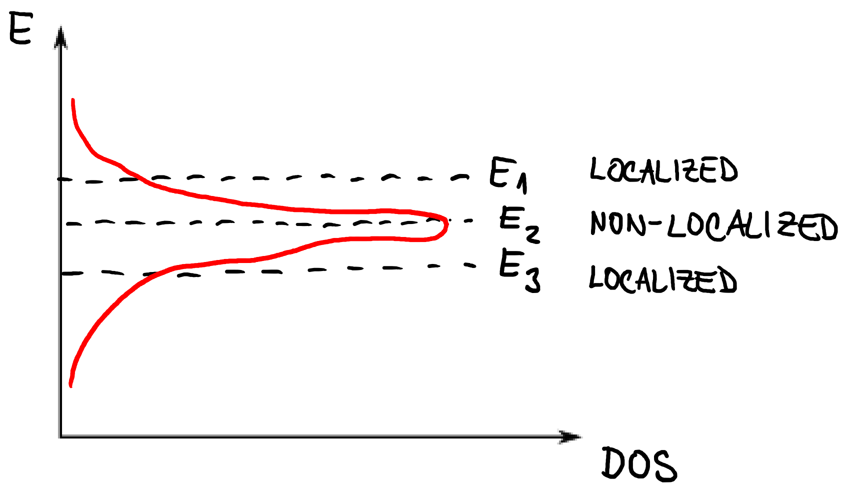
$\Rightarrow$ BROADENING OF LLs !



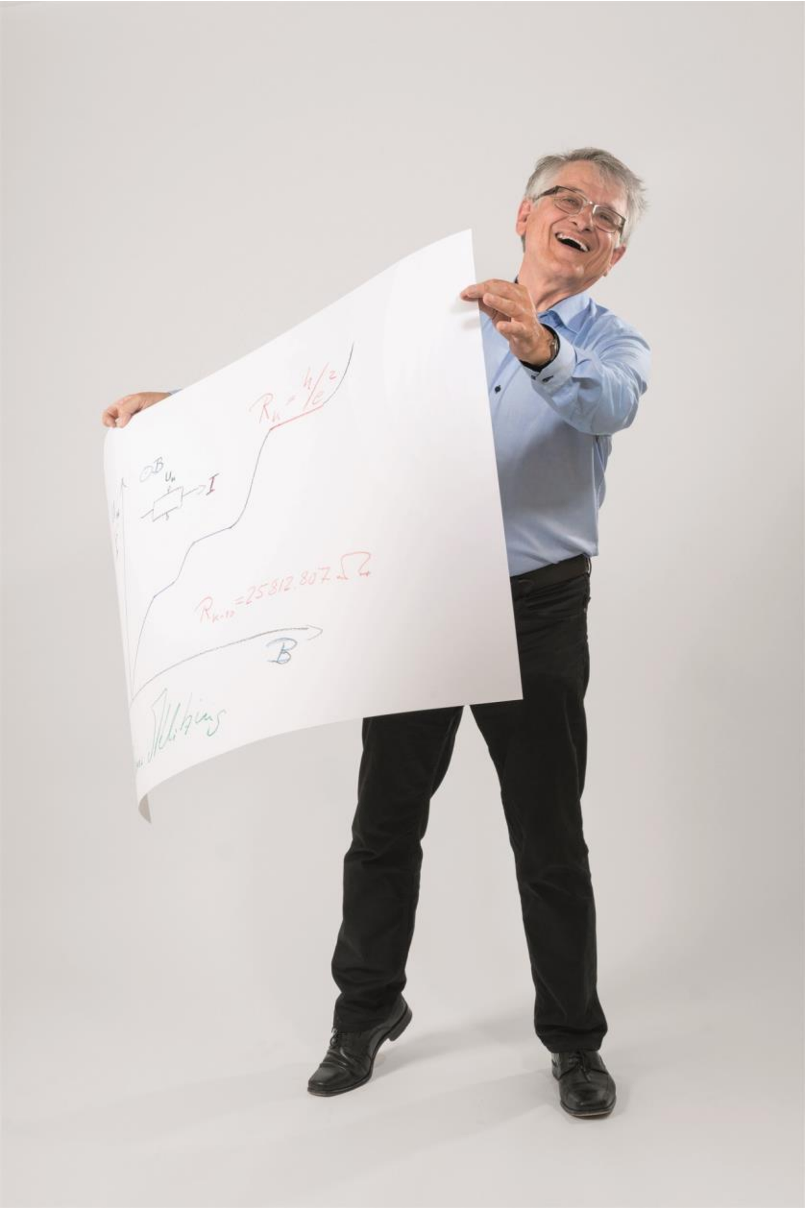
QUANTUM HALL EFFECT: LOCALIZED STATES







QUANTUM HALL EFFECT: CHECK OUT YOUR UNDERSTANDING


Klaus von Klitzing, Nobel prize for QHE in 1985

1. Can you measure your QHE @ home? What temperatures do you need if you are equipped with a Nd-based permanent magnet (1T @ surface) and your 2D material is made of GaAs ($m^* = 0{,}067\, m_0$)?

free electron mass

Help: LL spacing in E should be compared to $k_B T$

And what if you used a scotch tape and pencil and exfoliate a nice piece of graphene as the sample?

Help: Look @ the slide with spectroscopy of LL: graphene has different quantization of LL:

$E_M = v_F \sqrt{|2e\hbar B m|}$ $\quad v_F \sim 10^6$ m/s (Fermi velocity)

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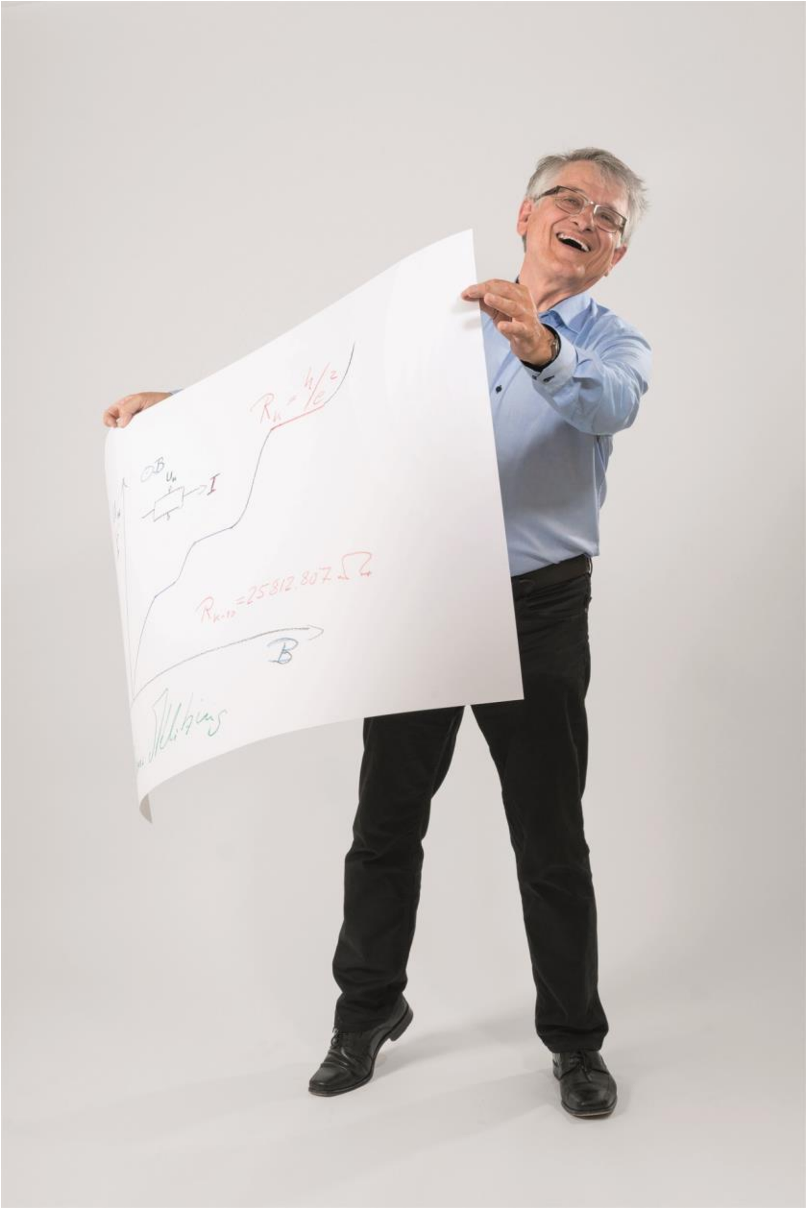

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time between 2 collisions

(2) Electrons are well localized in their LLs if their scattering time τ allows them to complete at least 1 orbit. Which of these materials would you choose to observe well visible quantum Hall plateaus (you still use the permanent magnet):

material	mobility μ ($cm^2/(Vs)$)
GaAs/AlGaAs QW	10^5
Si	1400
GaAs	8500
Al	13
Ag	60

Help: Condition is $\omega_c \tau > 1$. I remind that μ is related to τ.

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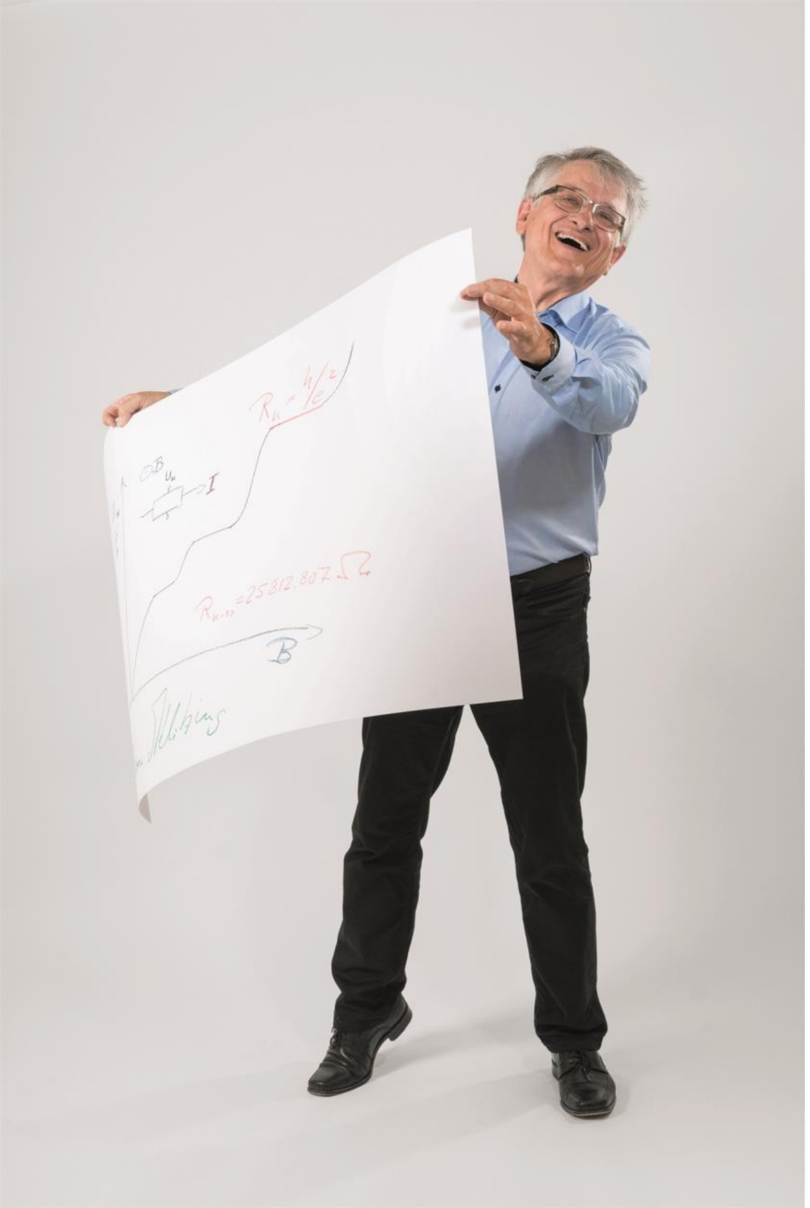

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(3) Why is everyone talking about 2D materials? Can you observe QHE in 3D bulk samples? Do you have a fundamental argument?

Help: Think about LLs in 3D. How do their cyclotron orbit look like?
And what about localized states?

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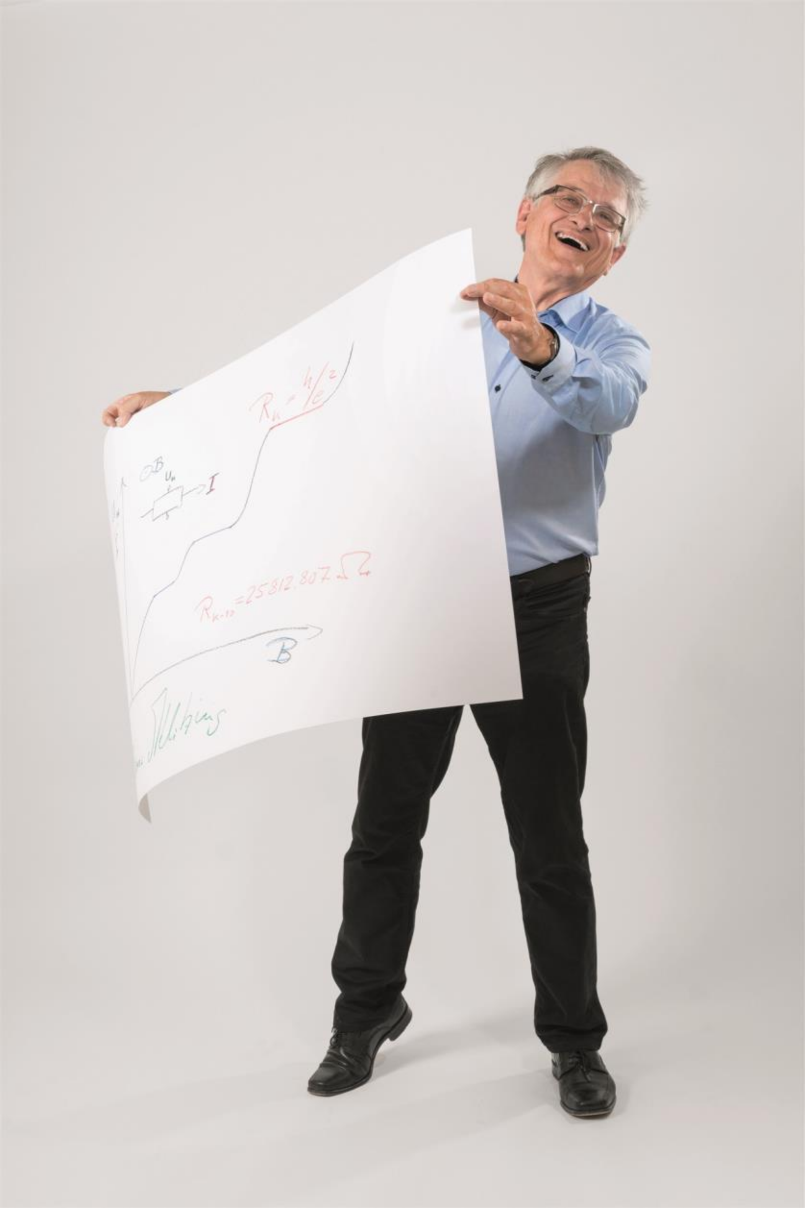

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If you got interested and wanna learn more just write me

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or google the following suggested topics:

- Shubnikov de Haas oscillations
- fractional quantum Hall effect
- Hofstadter's butterfly
- Laughlin gedanken experiment
- commensurability oscillations
- artificial graphene