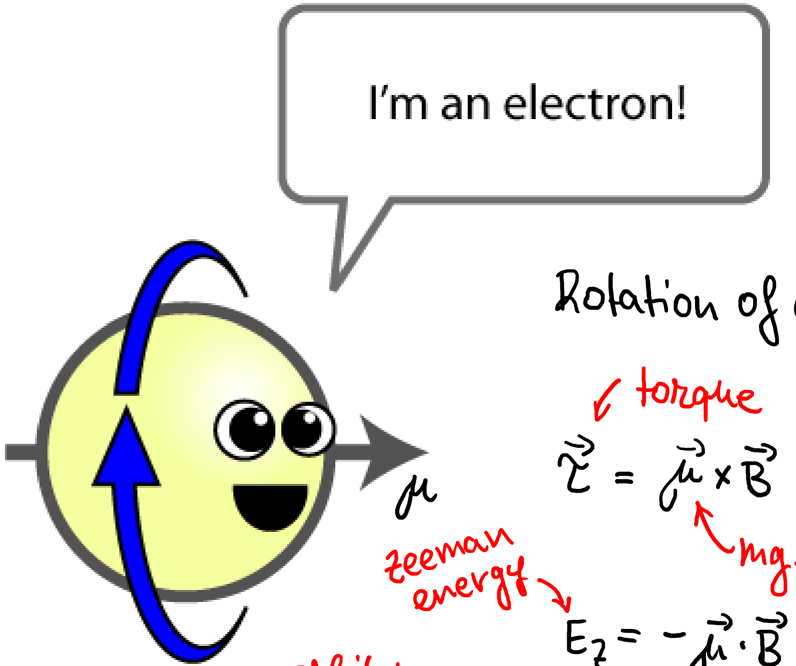
SPIN: INTRO



μ

Rotation of charged sphere?

torque

$\vec{\tau} = \vec{\mu} \times \vec{B}$

mg. moment

zeeman energy

$E_Z = -\vec{\mu} \cdot \vec{B}$

orbital momentum

spin momentum

$$\vec{\mu} = \frac{e}{2m}\vec{L} \rightarrow \frac{e}{2m} g \vec{S}$$

gyroscopic factor γ

Landé g-factor ~2

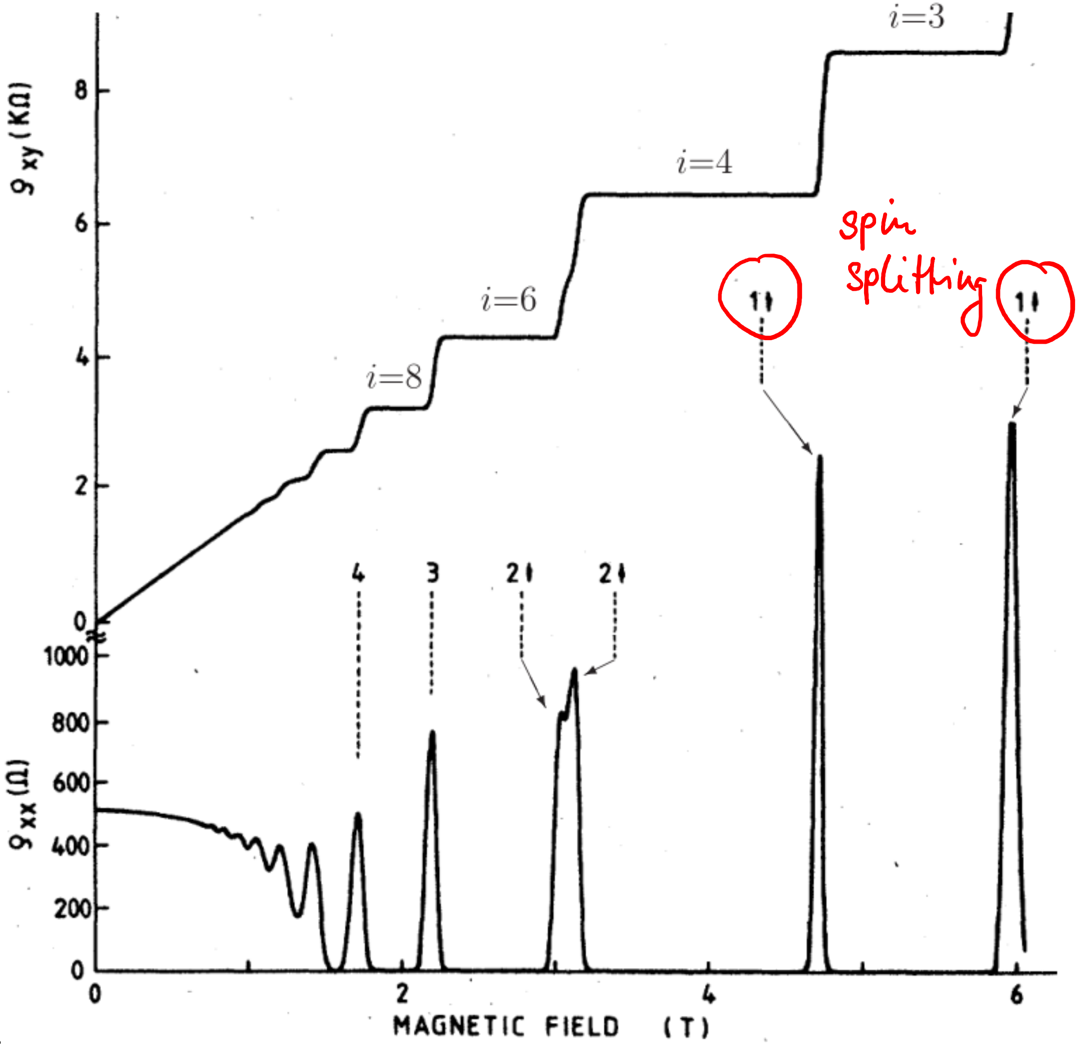
$|\vec{S}| = \frac{\sqrt{3}}{2}\hbar \qquad S_z = \frac{\hbar}{2}$

$\vec{S} = \vec{I}\omega \qquad I \propto ma^2$

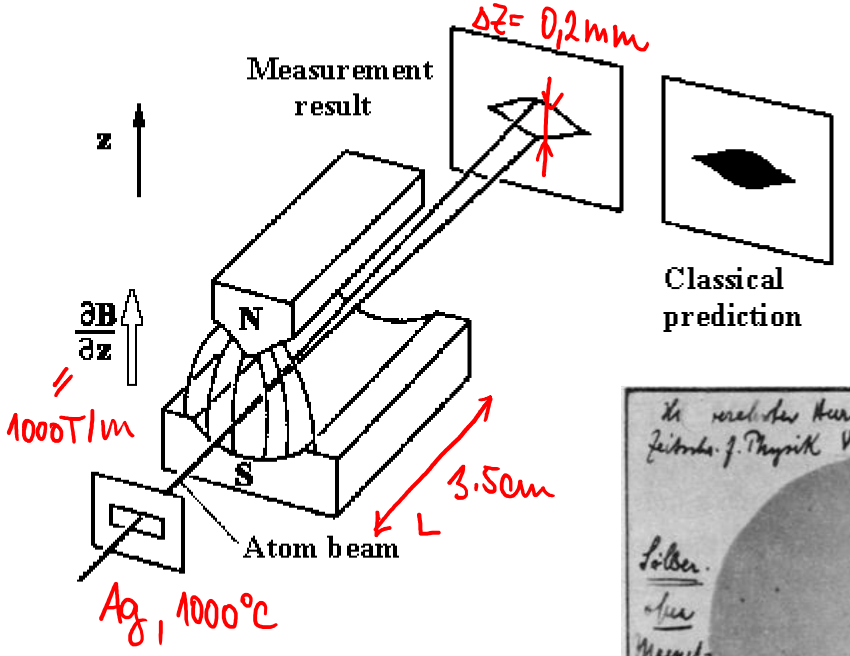
"orbital momentum of sphere"

$S \propto ma^2 \frac{v}{a}$

$a \sim 10^{-17} m, \quad m \sim 10^{-30}, \quad \hbar \sim 10^{-34}$ $\Big\}$ $v \sim 10^{13}\, m/s \gg c.$



SPIN: STERN-GERLACH EXPERIMENT



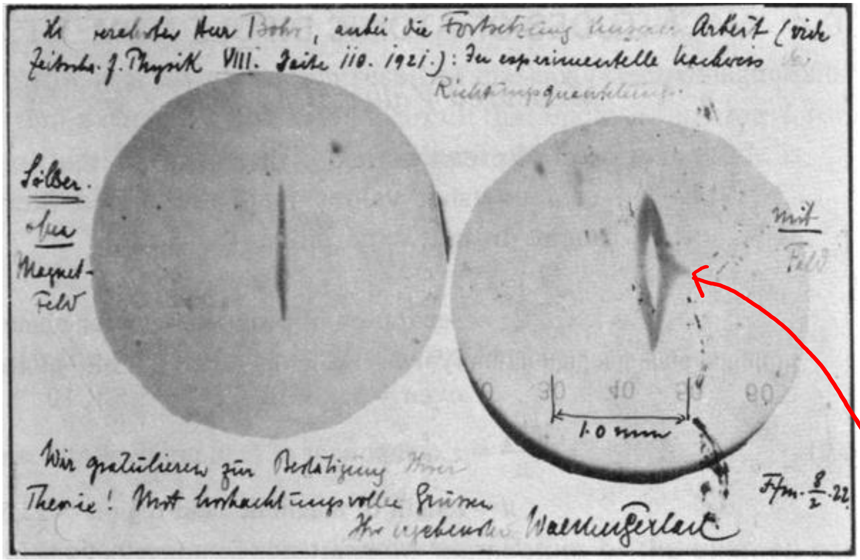
$$F = \frac{\partial E_z}{\partial z} \qquad E_z = \mu B_z$$

SI[1]	9.274 009 994(57) × 10^{-24}	$J \cdot T^{-1}$

$$\underbrace{\Delta z}_{2\cdot 10^{-4}} = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}\left[\frac{L}{v}\right]^2 = \pm \frac{\mu_B}{4}\frac{\overbrace{L^2}^{3{,}5\cdot 10^{-2}}}{\underbrace{E_K}_{k_B T}}\underbrace{\frac{\partial B_z}{\partial z}}_{10^3}$$

$k_B T$: 10^{-23}, 10^3

$\Rightarrow \mu_B \sim 7\text{-}8 \cdot 10^{-24}\ J/T$



Ag_2S (from cigars)

Ag : {Kr} $4d^{10}\ 5s^1$ $\quad L=0$

$\underline{\underline{S = 1}}$

Really good luck!

https://doi.org/10.1063/1.1650229

SPIN: QUANTUM MECHANICS OF SPIN

How to connect spin with QM?

- Empiric Pauli eq: $\underbrace{i\hbar \frac{\partial}{\partial t}\psi = \hat{H}\psi}_{\text{Schrödinger}} \rightarrow \hat{H} = \frac{1}{2m}(p - eA)^2 \mathbb{1} - \underbrace{\frac{e\hbar}{2m}\vec{\sigma}\cdot\vec{B}}_{\text{Pauli term}}$

Pauli matrices $\vec{\sigma} = \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]$

$\rightarrow$ no relativity :(

- Klein-Gordon eq: $\frac{E^2}{c^2} = |\vec{p}|^2 + m^2c^2$ & $\vec{p} \rightarrow i\hbar\vec{\nabla}\cdot$, $E \rightarrow i\hbar\frac{\partial}{\partial t}\cdot$ $\Rightarrow$ $\frac{\hbar^2}{c^2}\frac{\partial^2}{\partial t^2}\psi = \nabla^2\psi - m^2c^2\psi$ quadratic in t, r

$\hookrightarrow E = \pm\sqrt{\dots}$:(

- Dirac eq (linear): Let's take E… $\underbrace{i\hbar\frac{\partial}{\partial t}}_{E}\psi = (c\underbrace{\vec{\alpha}\cdot\vec{p}}_{p} + \underbrace{\vec{\beta}\, mc^2}_{mc^2})$ $/^2$ & compare to Klein Gordon

podm. $\beta^2 = \alpha_x^2 = \alpha_y^2 = \alpha_z^2 = 1$

$\alpha_i\beta + \beta\alpha_i = 0$ anticommut.

$\alpha_i\alpha_j + \alpha_j\alpha_i = 0 \quad (i \neq j)$

cannot be scalars but 4x4 (or 6x6…) matrices

$\rightarrow$ try $\vec{\sigma}$: $\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$, $\beta = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$

$\psi = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} \quad \psi^+ = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \psi^- = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}$

dim 4 is the price for linearity of Dirac eq.

SPIN: QUANTUM MECHANICS OF SPIN

$$\rightarrow \left[\begin{pmatrix} 0 & c\vec{\sigma}\vec{p} \\ c\vec{\sigma}\vec{p} & 0 \end{pmatrix} + \begin{pmatrix} mc^2 & 0 \\ 0 & -mc^2 \end{pmatrix}\right]\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} = E\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} - V\mathbb{1}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}$$

$$(E - mc^2 - V)\,\psi^+ = c\vec{\sigma}\vec{p}\,\psi^-$$

$$(\underbrace{E + mc^2}_{\sim 2mc^2} - V)\,\psi^- = c\vec{\sigma}\vec{p}\,\psi^+$$

REL. LIMIT ($E < mc^2$)

$$\psi^- = \frac{1}{2mc^2}\, c\vec{\sigma}\vec{p}\,\psi^+$$

$$\rightarrow E\psi^+ = \frac{1}{2m}(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{p})\,\psi^+ + mc^2\psi^+ + V\psi^+$$

$$(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$$

(1) $\vec{p}$, $\vec{p}$, $\vec{p}^{\,2}$, 0 $\Rightarrow \hat{H} = \frac{1}{2m}p^2 + mc^2 + V$ SCHRÖDINGER

(2) $\vec{p} - e\vec{A}$ $\Rightarrow \hat{H} = \frac{1}{2m}\left[(\vec{p} - e\vec{A}) + i\vec{\sigma}\cdot(\vec{p} - e\vec{A})\times(\vec{p} - e\vec{A})\right) + V + mc^2\big]$

$\vec{p}\times\vec{p} = 0$, $\vec{A}\times\vec{A} = 0$, $i\hbar\underbrace{\vec{\nabla}\times\vec{A}}_{\vec{B}}$ $(\vec{p} = -i\hbar\vec{\nabla}\cdot)$

$$-\underbrace{\frac{e\hbar}{2m}}_{=\mu_B}\vec{\sigma}\cdot\vec{B} + V + mc^2$$

μ_B Bohr's magneton

Zeeman interaction

$$\Rightarrow \boxed{\hat{H}_Z = \frac{g_s\mu_B}{\hbar}\vec{S}\cdot\vec{B}} = \frac{e}{m}\vec{S}\cdot\vec{B} \qquad \vec{S} = \frac{\hbar}{2}\vec{\sigma} \qquad g_s \approx 2 \text{ in vac.}$$

SPIN: QUANTUM MECHANICS OF SPIN

$$\rightarrow \left[\begin{pmatrix} 0 & c\vec{\sigma}\vec{p} \\ c\vec{\sigma}\vec{p} & 0 \end{pmatrix} + \begin{pmatrix} mc^2 & 0 \\ 0 & -mc^2 \end{pmatrix}\right]\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} = E\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} - V\mathbb{1}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}$$

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REL. LIMIT ($E < mc^2$)

$$\psi^- = \frac{1}{2mc^2}\,c\vec{\sigma}\vec{p}\,\psi^+$$

REARRANG.

NON-RELAT. LIMIT

$$\psi^- = \underbrace{\frac{1}{2mc}\,\vec{\sigma}\vec{p}\,\psi^+}_{\text{same as before}} - \underbrace{\frac{1}{2mc^2}\,(E - mc^2 - V)\,\psi^-}_{\text{new approx.}}$$

and again to 1st eq.

we get: $\frac{c}{2m^2c^3}\,\vec{\sigma}\cdot\vec{p}\,(E - mc^2 - V)\,\vec{\sigma}\cdot\vec{p}$

$V \to eV$

$$\boxed{\hat{H}_{SO} = \frac{e}{2mc^2}\,\vec{S}\cdot(\vec{v}\times\vec{E})}$$

$\frac{\hbar}{2}\vec{\sigma}$ $\quad$ $\frac{\vec{p}}{m}$ $\quad$ ∇V

for spherical V

$$= \frac{1}{2m^2c^2}\,S\cdot\frac{1}{r}\frac{dV}{dr}\,\underbrace{(\vec{r}\times\vec{p})}_{L} = \xi\cdot\vec{S}\cdot\vec{L}$$

Spin - orbit interaction

$(\sigma\cdot p)\,V\,(\sigma\cdot p) =$

$= [\sigma\cdot(pV)](\sigma\cdot p) =$ identity from NON-REL. LIM.

$= p\cdot V\cdot p + i\sigma\cdot(pV\times p) \propto$ $\qquad p \to -i\hbar\nabla$.

$\propto \nabla V\cdot p - \underbrace{\sigma\cdot(\nabla V\times p)}_{\to \text{SOI}}$

SPIN : QUANTUM MECHANICS OF SPIN

$$\rightarrow \left[\begin{pmatrix} 0 & c\vec{\sigma}\vec{p} \\ c\vec{\sigma}\vec{p} & 0 \end{pmatrix} + \begin{pmatrix} mc^2 & 0 \\ 0 & -mc^2 \end{pmatrix}\right]\begin{pmatrix}\psi^+ \\ \psi^-\end{pmatrix} = E\begin{pmatrix}\psi^+ \\ \psi^-\end{pmatrix} - V\mathbb{1}\begin{pmatrix}\psi^+ \\ \psi^-\end{pmatrix}$$

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and again to 1st eq.

we get : $\frac{c}{2m^2c^3}\,\underline{\vec{\sigma}\cdot\vec{p}}\,(E - mc^2 \underline{-V})\,\underline{\vec{\sigma}\cdot\vec{p}}$

$V \rightarrow eV$

$$\boxed{\hat{H}_{SO} = \frac{e}{2mc^2}\vec{S}\cdot(\vec{v}\times\vec{E})}$$

$\frac{\hbar}{2}\vec{\sigma}$ $\quad \frac{\vec{p}}{m}$ $\quad \nabla V$

$$\longleftrightarrow \quad \hat{H}_z = \frac{e}{m}\vec{S}\cdot\vec{B}$$

missing 2

$$\Rightarrow \boxed{\vec{B}_{SOI} = \frac{\vec{v}\times\vec{E}}{c^2}}$$

$(\sigma\cdot p)V(\sigma\cdot p) =$

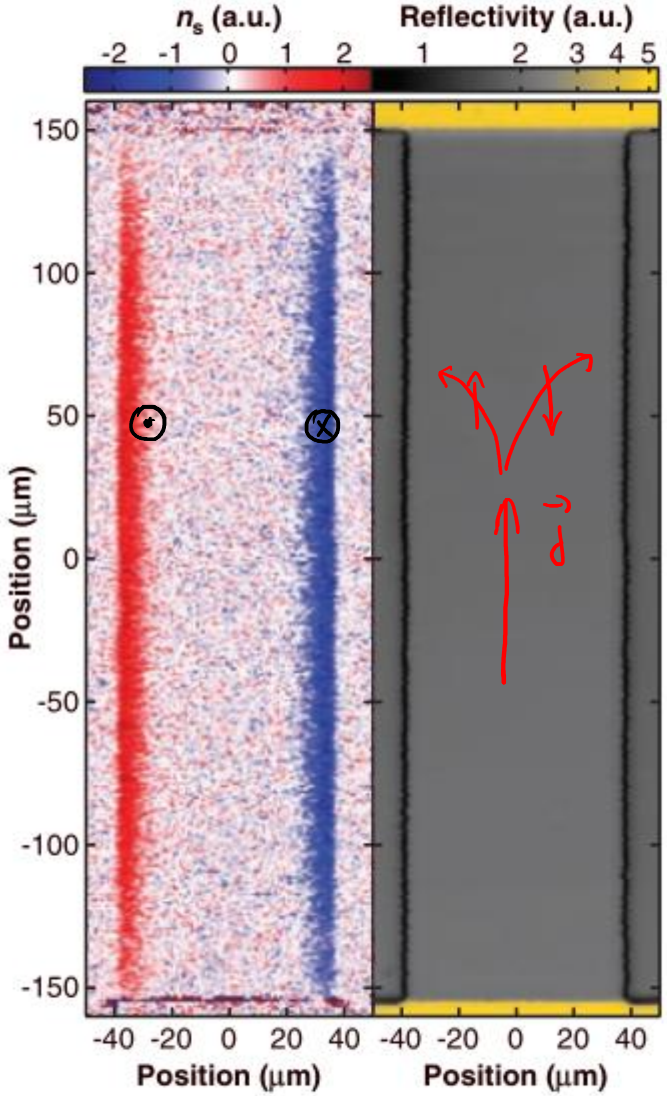
$= [\sigma\cdot(pV)](\sigma\cdot p) =$ identity from NON-REL.LIM.

$= p\cdot V\cdot p + i\sigma\cdot(pV\times p) \propto$ $\quad p\rightarrow -i\hbar\nabla$.

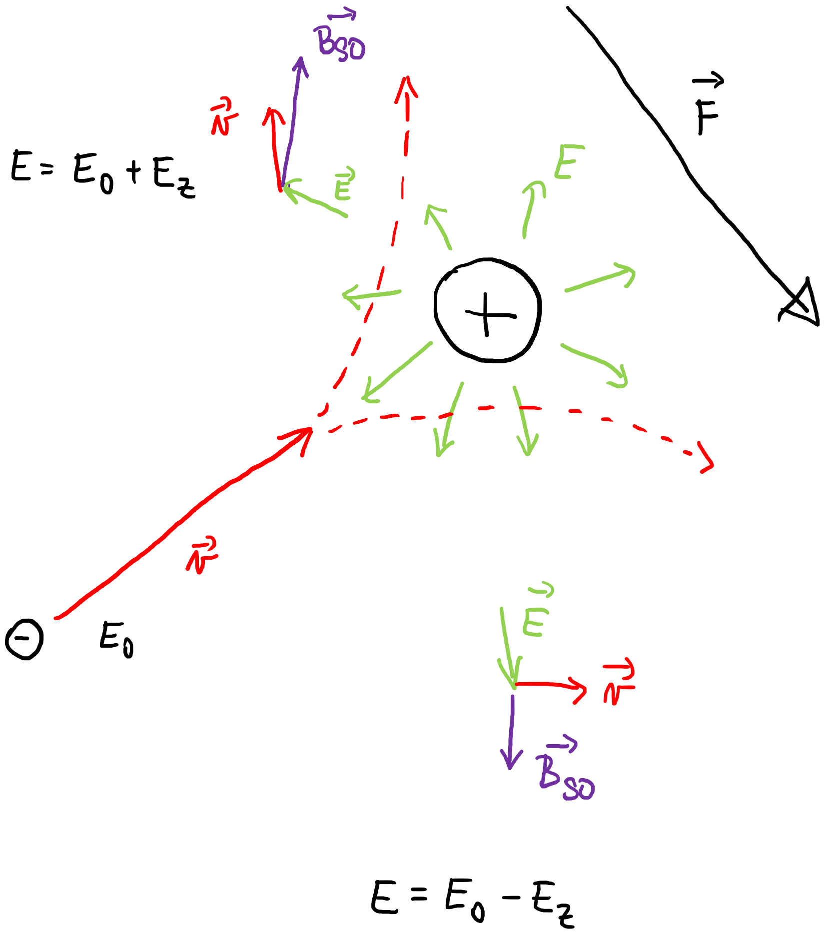
$\propto \nabla V\cdot p - \underbrace{\sigma\cdot(\nabla V\times p)}_{\rightarrow\ SOI}$

SPIN : QUANTUM MECHANICS OF SPIN

⇒ Let's use it immediately : $\vec{B} = \frac{\vec{v} \times \vec{E}}{c^2}$



Kato et al., Science 2004



$\vec{F} = -\nabla E$

Force ↔ SPIN HALL EFFECT

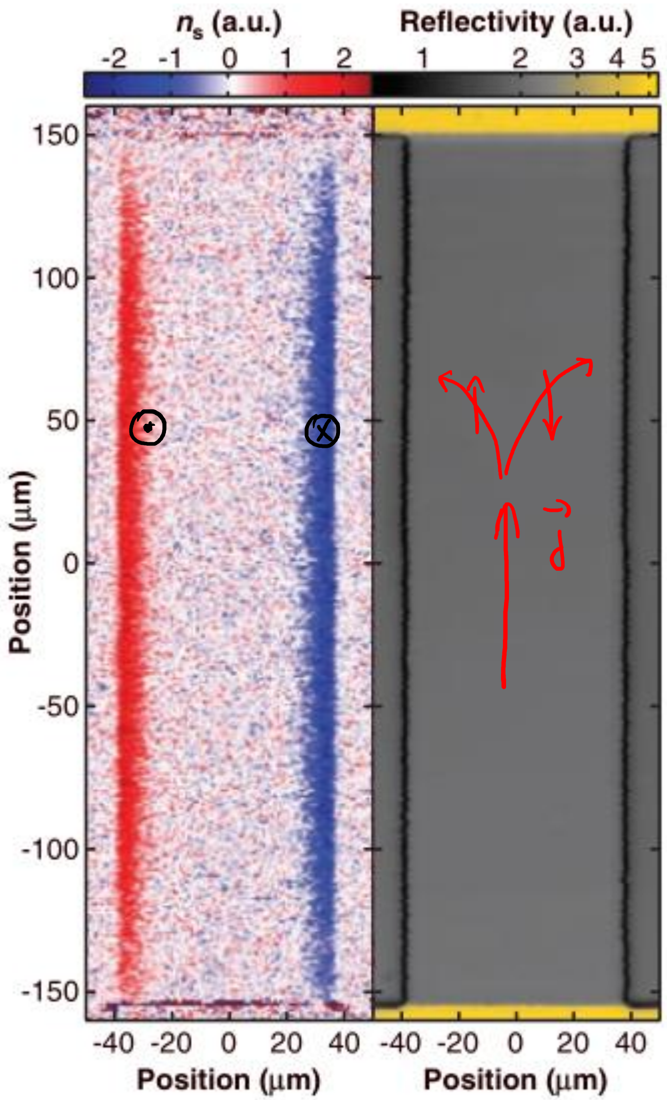
$H_z = \frac{g_s \mu_B}{\hbar} \vec{S} \cdot \vec{B}$ $\vec{S} = \frac{\hbar}{2} \vec{\sigma}$ $g_s \approx 2$

$H_z = \mu_B \vec{\sigma} \cdot \vec{B} \Rightarrow E_z = \pm \mu_B B$

eigen values ± 1

SPIN : ~~QUANTUM MECHANICS OF SPIN~~ SPIN HALL EFFECT

⇒ Let's use it immediately : $\vec{B} = \dfrac{\vec{v}\times\vec{E}}{c^2}$



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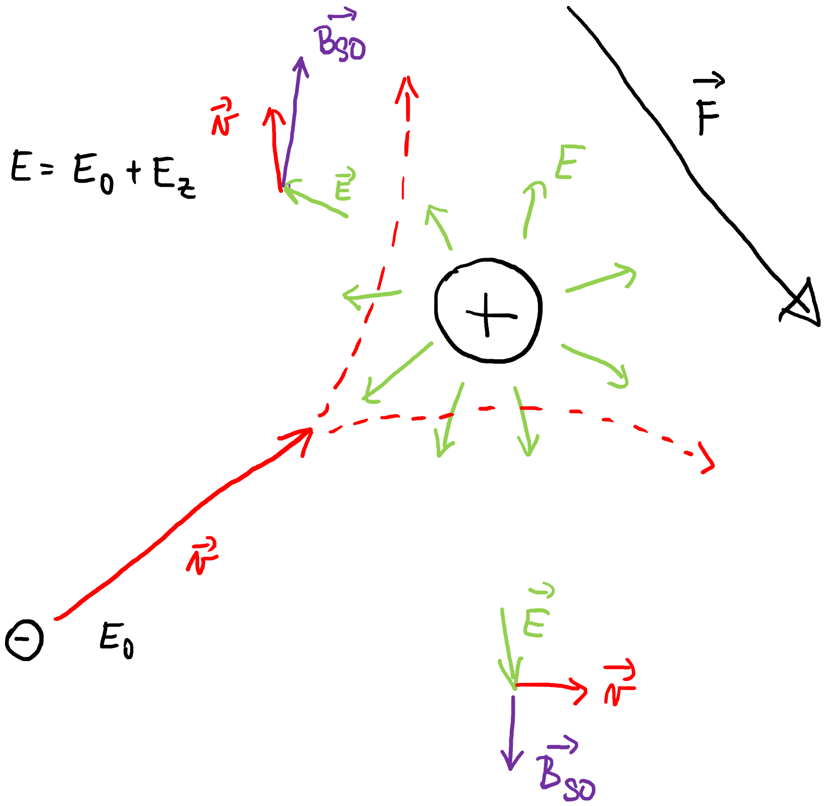


$E = E_0 + E_z$

$E = E_0 - E_z$

$\vec{F} = -\nabla E$

Force ⟷ SPIN HALL EFFECT

$H_z = \dfrac{g_s \mu_B}{\hbar}\,\vec{S}\cdot\vec{B}$ $\vec{S} = \dfrac{\hbar}{2}\vec{\sigma}$ $g_s \approx 2$

$H_z = \mu_B\,\vec{\sigma}\cdot\vec{B} \Rightarrow E_z = \pm\,\mu_B B$

eigen values ± 1

SPIN: PRECESSION

reminder: $\hat{H} = \underbrace{\frac{(\vec{p}-e\hat{A})^2}{2m}}_{\text{KINETIC + LANDAU LEV.}} + \underbrace{\frac{g_s \mu_B}{\hbar}\vec{S}\cdot\vec{B}}_{\text{ZEEMAN (or SOI)}} \rightarrow E = \frac{\hbar^2 k^2}{2m} + \hbar\omega_c(n+1/2) + g_s\mu_B B_z \cdot s \qquad s = \pm 1/2$

$\hat{H}\psi = E\psi \qquad \psi = \varphi(x,t)\,\chi(t)$

SPINOR $\begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix}$

For $B = (0,0,B)$

$\sigma_z = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $\begin{pmatrix}1\\0\end{pmatrix}$ & $\begin{pmatrix}0\\1\end{pmatrix}$ are eigenstates

$\downarrow$ 1 $\qquad \downarrow$ −1 eigen value

Larmor freq.

$\boxed{E_z = \pm\frac{1}{2}\hbar\omega_L} \qquad \boxed{\omega_L = \gamma B = \frac{g_s e}{2m}B}$

$\psi = ?$ $\qquad U(t) = e^{-i\frac{E}{\hbar}t}$

$$\chi_{\substack{1,2\\ +,-}}(t) = \begin{cases}\begin{pmatrix}1\\0\end{pmatrix} e^{-i\frac{\omega_L}{2}t}\\ \begin{pmatrix}0\\1\end{pmatrix} \underbrace{e^{+i\frac{\omega_L}{2}t}}_{=e^-,\,e^+}\end{cases}$$

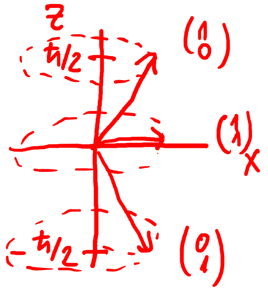
We oscillate between eigenstates

$\chi(t) = c_1\chi_1 + c_2\chi_2$

$\langle A\rangle = \psi^\dagger A \psi$

$\Rightarrow$ state χ^+: $\langle S_z\rangle = \frac{\hbar}{2}\chi^{+\dagger}\sigma_z\chi^+ = \frac{\hbar}{2}(1\cdot e^+, 0)\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1e^-\\0\end{pmatrix} = \frac{\hbar}{2}(e^+,0)(e^-,0) = +\frac{\hbar}{2}$

χ^-: $\langle S_z\rangle = \chi^{\dagger}\ \chi^- = \ldots\ldots = \frac{\hbar}{2}(0,e^-)(0,-e^+) = -\frac{\hbar}{2} \qquad \langle S_x\rangle = 0$



? $\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(\chi_+ + \chi_-)$: $\langle S_z\rangle = \frac{1}{2}\frac{\hbar}{2}(e^+,e^-)\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}e^-\\e^+\end{pmatrix} = \frac{\hbar}{4}(e^+,e^-)(e^-,-e^+) = \frac{\hbar}{4}(1-1) = 0$

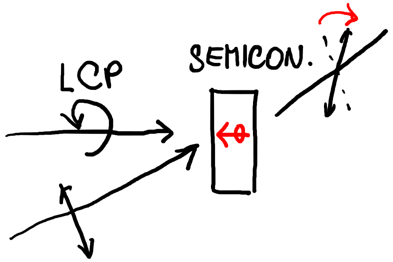
$\langle S_x\rangle = \frac{1}{2}\frac{\hbar}{2}(e^+,e^-)\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}e^-\\e^+\end{pmatrix} = \frac{\hbar}{4}(e^+,e^-)(e^+,e^-) = \frac{\hbar}{2}\frac{e^{2+}+e^{2-}}{2} = \frac{\hbar}{2}\cos\omega_L t$

SPIN: PRECESSION

In GaAs: $f_L = \frac{\omega_L}{2\pi} \sim 10\,\text{GHz} = 100\,\text{ps}$ Very fast

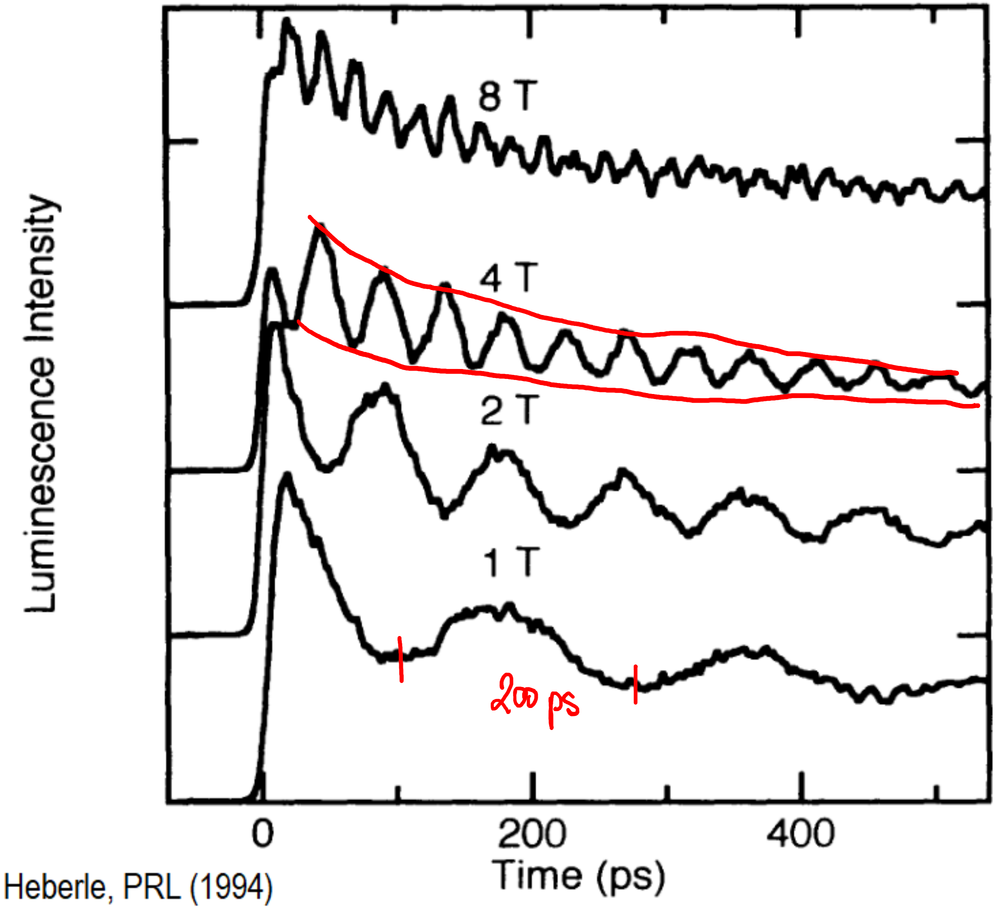
$g_S = -0{,}44$ How to measure both?

} OPTICS

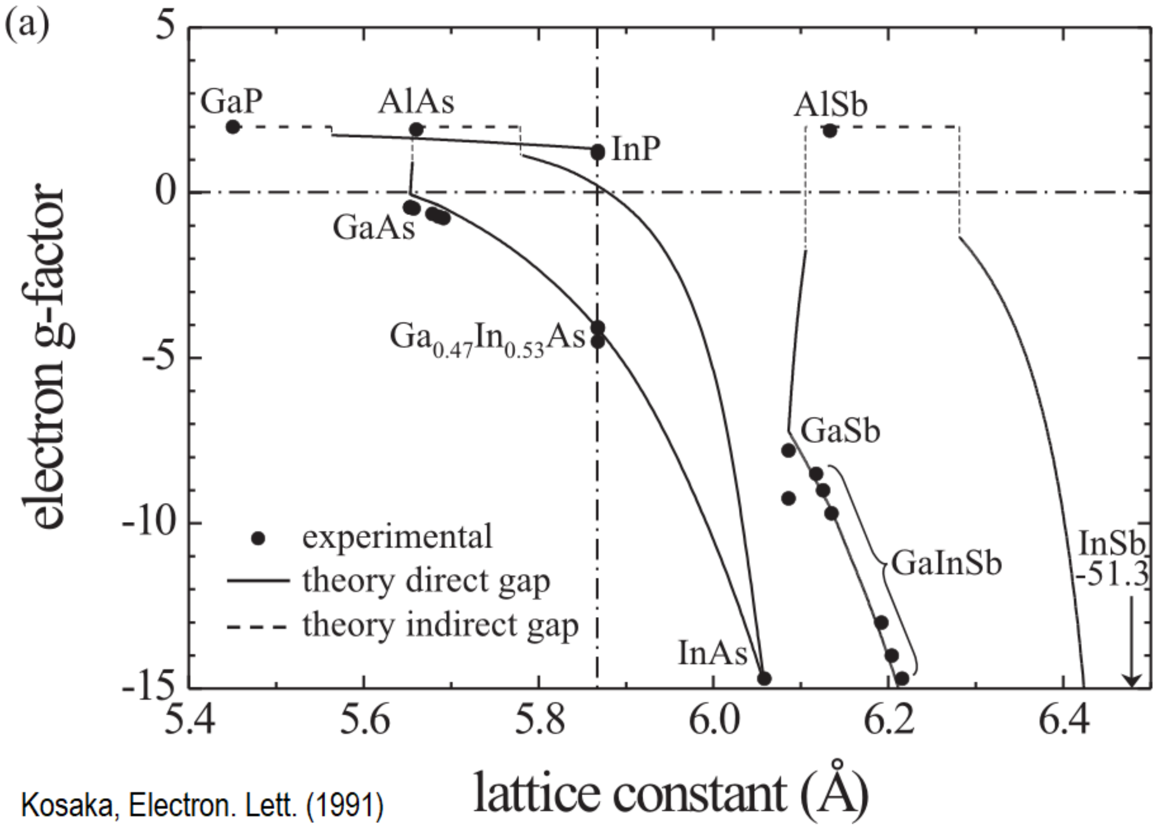


MAGNETO OPTICS

& OPTICAL SPIN INJECTION



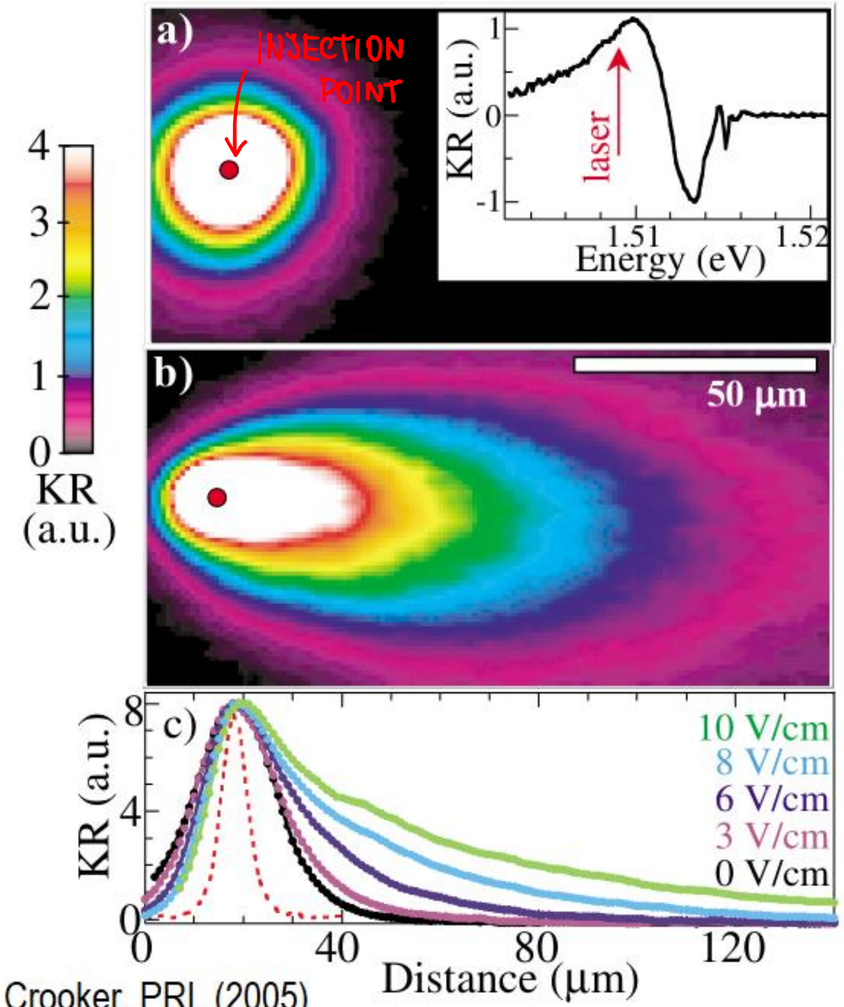
Heberle, PRL (1994)



Kosaka, Electron. Lett. (1991)

+ depends on T, confinement...

SPIN: DRIFT-DIFFUSION OF SPINS



Crooker, PRL (2005)

	METAL	SEMICOND.
l_s	10-100 nm	1-10's µm
τ_s	10's fs	100's ps - 10's ns

→ motion of spins = spin current

$$\vec{j}_{\hat{s}} = \frac{\hbar}{2} m \vec{n} P_{\hat{s}} = \vec{n} S_{\hat{s}}$$

0-100% spin polar.

density of spin moments

$$\vec{j}_{\hat{s}} = -\mu \vec{E} S_{\hat{s}} - D_s \nabla S_{\hat{s}}$$ Fick's law

$$\underbrace{\frac{\partial S_{\hat{s}}}{\partial t}}_{\text{temp. change}} + \underbrace{\nabla \vec{j}_{\hat{s}}}_{\text{flow}} + \underbrace{\vec{\omega}_L \times \vec{S}}_{\text{precession}} + \underbrace{\frac{S_{\hat{s}}}{\tau_s}}_{\text{relaxation}} = 0$$ Continuity eq.

Ex. (1) $\frac{\partial S}{\partial t} = 0,\ E = 0,\ \omega_L = 0 \Rightarrow -D_s \nabla^2 S(r) + \frac{S(r)}{\tau_s} = 0 \rightarrow S(r) \propto e^{-r/l_s}$

$$l_s = \sqrt{D_s \tau_s}$$

(2) $\frac{\partial S}{\partial t} = 0,\ \omega = 0,\ D_s = 0 \Rightarrow -\mu E \nabla S = -\frac{S}{\tau_s} \rightarrow S(r) = e^{-r/(\mu E \tau_s)}$

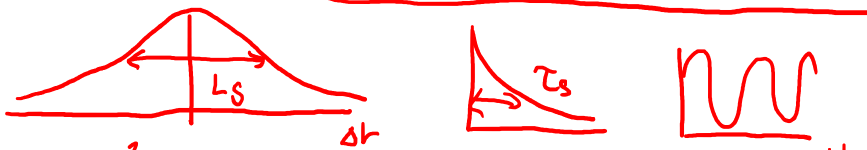
$$l_s = v_D \tau_s$$

$$\Downarrow$$

$$\boxed{S(\Delta r, \Delta t) = \exp\left(-\frac{(\Delta r + \mu E \Delta t)^2}{4 D_s \Delta t}\right) \exp\left(-\frac{\Delta t}{\tau_s}\right) \cos \omega_L t \cdot \frac{1}{D_s \Delta t}}$$

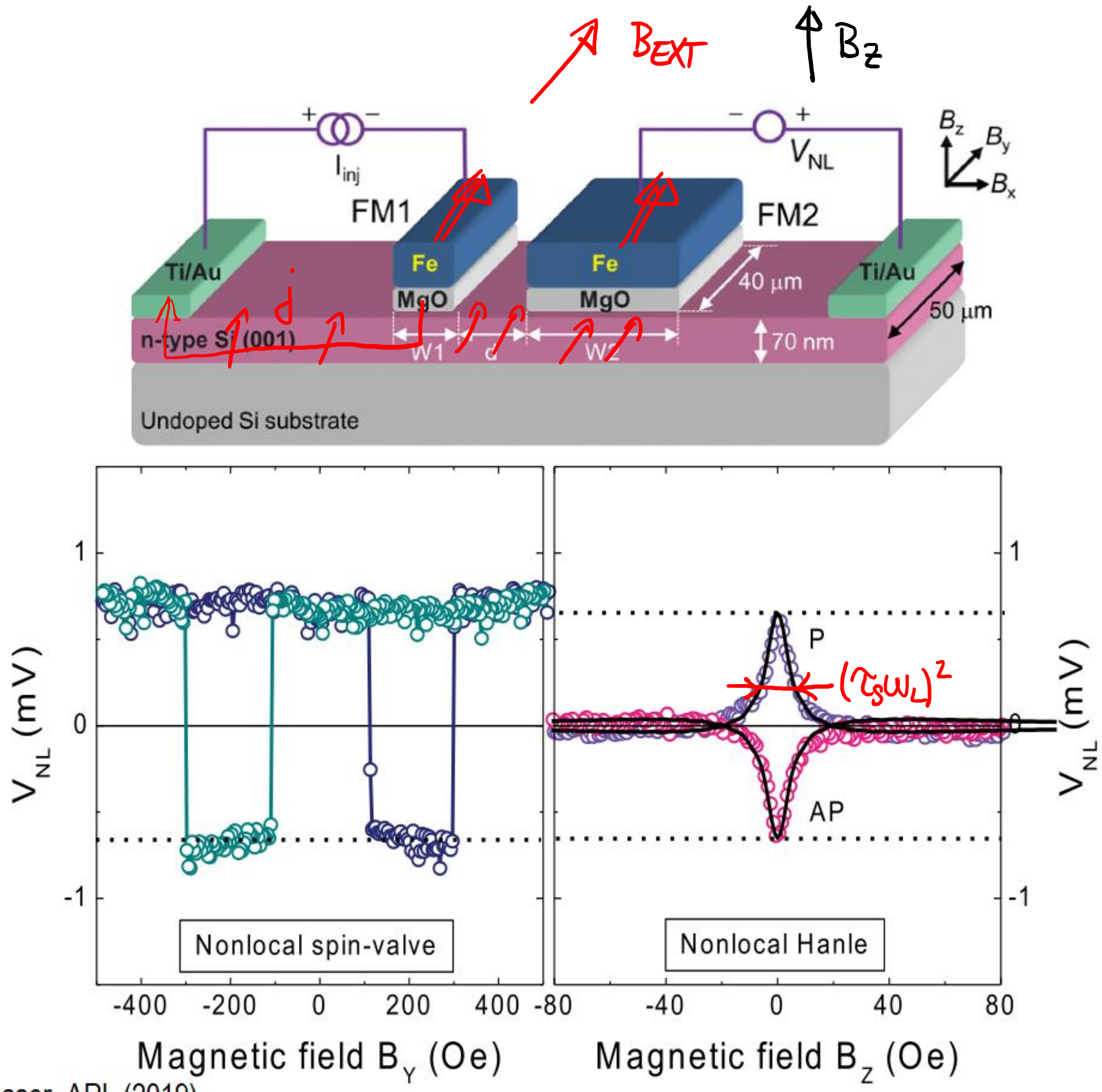
dynamic normal.

$$\frac{\partial S}{\partial t} = D \frac{\partial^2 S}{\partial x^2} \rightarrow S = \frac{1}{\sqrt{t}} e^{\frac{x^2}{4Dt}}$$



$$L_s^2 = 4 D_s \Delta t$$

SPIN: HANLE EFFECT



Spiesser, APL (2019)

How to measure D_s, τ_s without optical ultrafast resolution?

1. measure L_s: just increase distance of electrodes d.
2. vary B:

$$V(B_\perp) \propto \int_0^\infty S(\Delta r, \Delta t)\, d\Delta t$$

ignore diff. term

$$= \frac{\tau_s}{(\tau_s \omega_L)^2 + 1}$$

Halfwidth

SPIN: RELAXATION

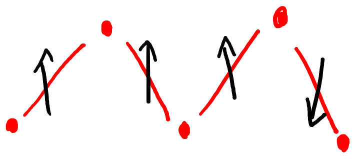
$S(t) = S_0 \, e^{-t/\tau_s}$?

From spin interactions : ① with $B(t)$, mg. atoms
② with B_{SO}

Ⓐ ELLIOTT - YAFET :

$$\Psi_{k\uparrow} = \left[a_k \begin{pmatrix}1\\0\end{pmatrix} + b_k \begin{pmatrix}0\\1\end{pmatrix} \right] e^{ikr}$$

$$|a_k| \gg |b_k|$$



$\sim 10^4 - 10^5$ per 1 flip

$$\boxed{\Gamma_S = \frac{1}{\tau_S} \propto \frac{1}{\tau_p} (\Delta g)^2}$$

k-vector relax.

Metals ($\tau_p \downarrow$, centrosymm.)

Semimetals (small gap)

Ⓑ DYAKONOV - PEREL : $\vec{B}_{SO} \propto \vec{v} \times \vec{E}$

$$\omega_{SO} = \frac{g \mu_B}{\hbar} B_{SO}(\vec{k})$$

spin precesses untie around $\bar{\omega}$ they get scattered → random walk

$\delta\phi = \bar{\omega}\,\tau_p$ phase change on 1 step

$\phi(t) = \delta\phi \sqrt{t/\tau_p}$ phase after t/τ_p steps

$$\phi(\tau_s) = 1 \Rightarrow \phi(\tau_s) = \delta\phi \sqrt{\tau_s/\tau_p} = \bar{\omega}\sqrt{\tau_p \tau_s} = 1$$

$$\boxed{\Gamma_S = \frac{1}{\tau_S} = \bar{\omega}^2 \tau_p}$$

Semicond. (large gap, structural or crystalline asymm.)
τ_p large

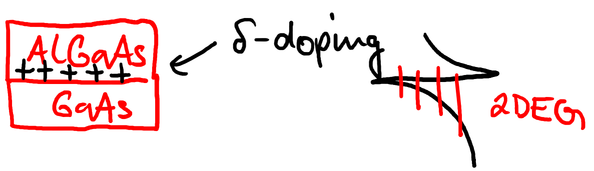
→ designing of τ_s in QWs

SPIN: SOI AND EFFECTIVE FIELD

SOI: $H_{SO} = \frac{e}{2mc^2}\,\frac{\hbar}{2}\vec{\sigma}\cdot(\vec{v}\times\vec{\nabla}V) \;\propto\; \nabla\vec{V}\cdot(\vec{\sigma}\times\vec{k})$ $\qquad m\vec{v} = \vec{p} = \hbar\vec{k}$

$\hookrightarrow$ complicated

IN 2D



or gating $\Rightarrow$ asymmetry in $\hat{z}$, $B=0 \Rightarrow H_Z = 0$

$$\boxed{\hat{H} = H_0 + H_{SO} = \frac{\hbar^2k^2}{2m} + \alpha(\nabla V)\left[\sigma_x k_y - \sigma_y k_x\right]}$$

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad \varepsilon_k = \frac{\hbar^2k^2}{2m}$ $\quad k_\pm = k_x \pm i k_y$

$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad \Rightarrow \hat{H} = \begin{pmatrix} \varepsilon_k & i\alpha k_- \\ -i\alpha k_+ & \varepsilon_k \end{pmatrix}$

SCHR. $\quad H\Psi = E\Psi \quad \Psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$ spinor $\quad \begin{pmatrix} \varepsilon_k - E & i\alpha k_- \\ -i\alpha k_+ & \varepsilon_k - E \end{pmatrix}\begin{pmatrix} \varphi \\ \chi \end{pmatrix} = 0$

$\rightarrow$ Eigen values: $\det = (\varepsilon_k - E)^2 - \alpha^2 k^2 = 0 \Rightarrow \boxed{E_{1,2} = \varepsilon_k \mp \alpha k}$ $\quad$ Shifted in k-space ... Hmm

$\rightarrow$ Eigen functions: $E_1 = \varepsilon_k - \alpha k$: $\chi_1 = \frac{i\alpha k_+}{\varepsilon_k - E_1}\varphi_1 = \frac{ik_+}{k}\varphi_1 \Rightarrow \boxed{\chi_1 = \begin{pmatrix} \varphi_1 \\ \chi_1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \frac{ik_+}{k} \end{pmatrix}}$

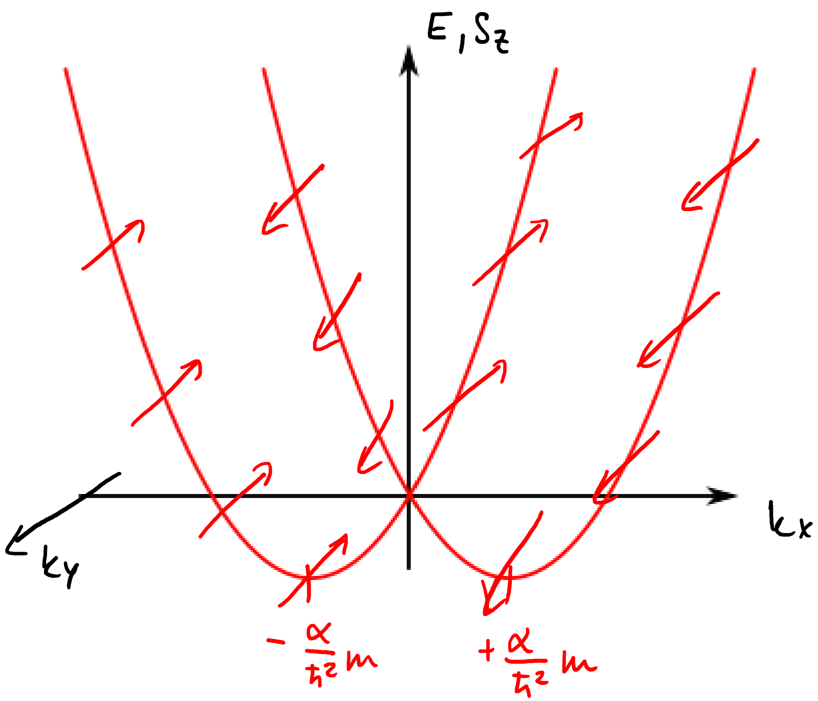
$E_2 = \varepsilon_k + \alpha k$: $\chi_2 = \ldots\ldots \Rightarrow \boxed{\chi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{ik_-}{k} \\ 1 \end{pmatrix}}$

SPIN: SOI AND EFFECTIVE FIELD

RASHBA FIELD

$\boxed{1D}$ $k_y = 0,\ k_x \neq 0$ $\quad E_{1,2} = \frac{\hbar^2 k_x^2}{2m} \mp \alpha k_x = k_x\left(\frac{\hbar^2 k_x}{2m} \mp \alpha\right)$ ← parabola shifted in k_x

$k_\pm = k_x$ $\quad \underbrace{\psi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} \qquad \psi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}i\\ 1\end{pmatrix}}_{\text{eigen functions of } \sigma_y\,!}$



$e^{i\frac{E}{\hbar}t}$

→ Let's add a spin in a mixed state:

$\psi_1(t) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} e^{i\frac{-\alpha k_x}{\hbar}t}$ $\quad e^-$

$\psi_2(t) = \frac{1}{\sqrt{2}}\begin{pmatrix}i\\ 1\end{pmatrix} e^{i\frac{+\alpha k_x}{\hbar}t}$ $\quad e^+$

$\psi = \psi_1 + \psi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}e^- + i e^+\\ i e^- + e^+\end{pmatrix}$ $\quad \langle A\rangle = \psi^\dagger A \psi$

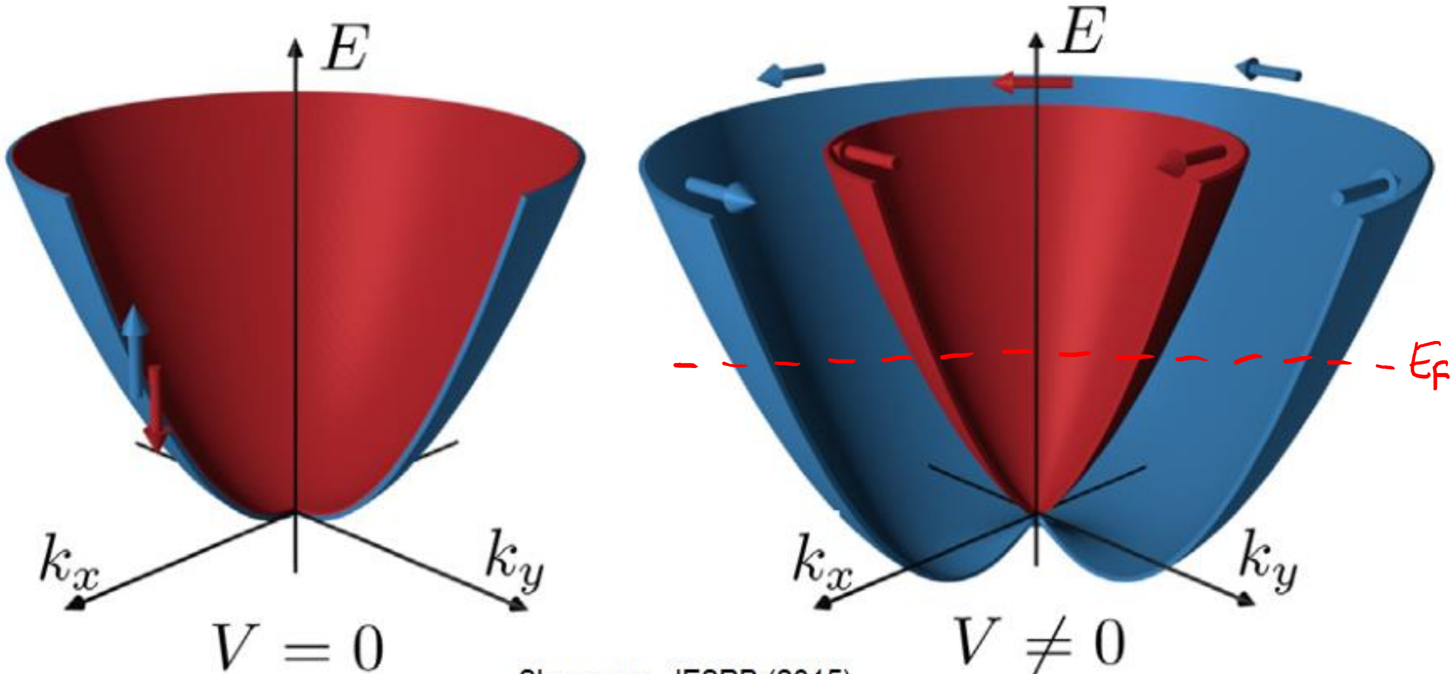
$\Delta E = g_s \mu_B B = \hbar\omega_{SO} = 2\alpha k_x$

$\Rightarrow \boxed{B_{SO} = \frac{2\alpha}{g\mu_B} k_x}$

ψ in $\langle S_z\rangle$: $\langle S_z\rangle = \frac{\hbar}{2}\frac{1}{2}\left(e^+ - ie^-, -ie^+ + e^-\right)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}e^- + ie^+\\ ie^- + e^+\end{pmatrix} = \frac{\hbar}{4}\left(e^+ - ie^-, -ie^+ + e^-\right)\left(e^- + ie^+, -ie^- - e^+\right) =$

$= \frac{\hbar}{4}\left[1 + ie^{2+} - ie^{2-} + 1 - 1 + ie^{2+} - ie^{2-} - 1\right] = \hbar\,\frac{e^{2+} - e^{2-}}{2i} = \hbar \sin\omega_{SO}t$ → $\boxed{\omega_{SO} = 2\frac{\alpha k_x}{\hbar}}$ $\propto E$

SPIN: SOI AND EFFECTIVE FIELD

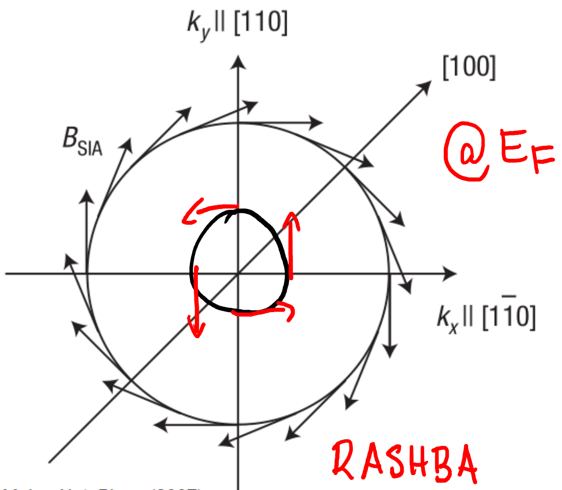


Shanavas, JESRP (2015)

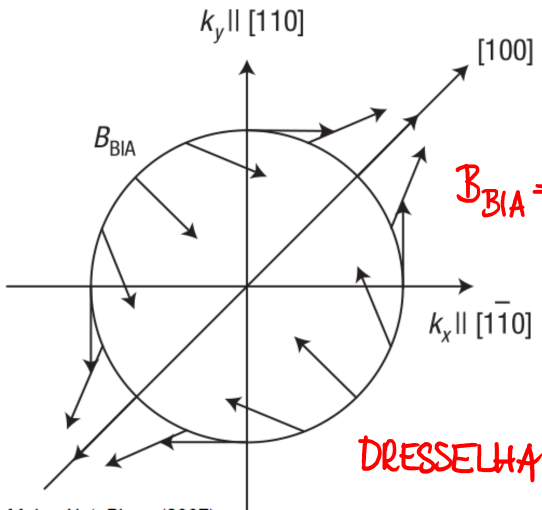
$$B_{SIA} = \frac{2\alpha}{g\mu_B}\begin{pmatrix} k_y \\ -k_x \end{pmatrix}$$

$$\text{phase}: \; e^{i\omega_{so}t} = e^{i\frac{2}{\hbar}k_y t} \rightarrow e^{i \ldots \alpha \hat{y}}$$

$$\hookrightarrow \frac{m}{\hbar} v_y = \frac{m}{\hbar}\frac{\vec{y}}{t}$$



Meier, Nat. Phys. (2007)



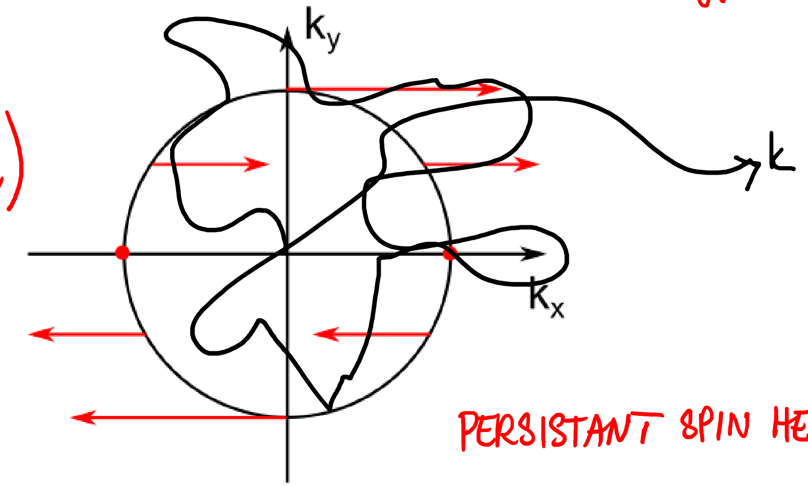
$$B_{BIA} = \frac{2\beta}{g\mu_B}\begin{pmatrix} k_y \\ k_x \end{pmatrix}$$

DRESSELHAUS

Meier, Nat. Phys. (2007)

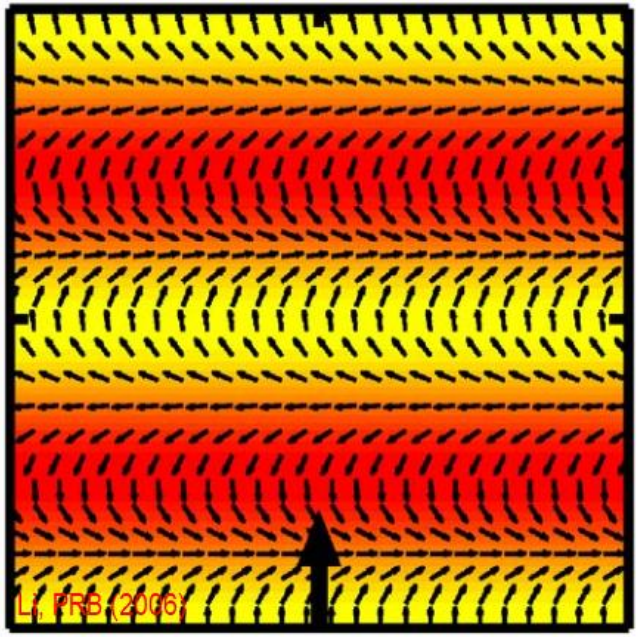
FOR $\alpha = \beta$

$$B_{PSH} = \frac{2\alpha}{g\mu_B}\begin{pmatrix} k_y \\ 0 \end{pmatrix}$$



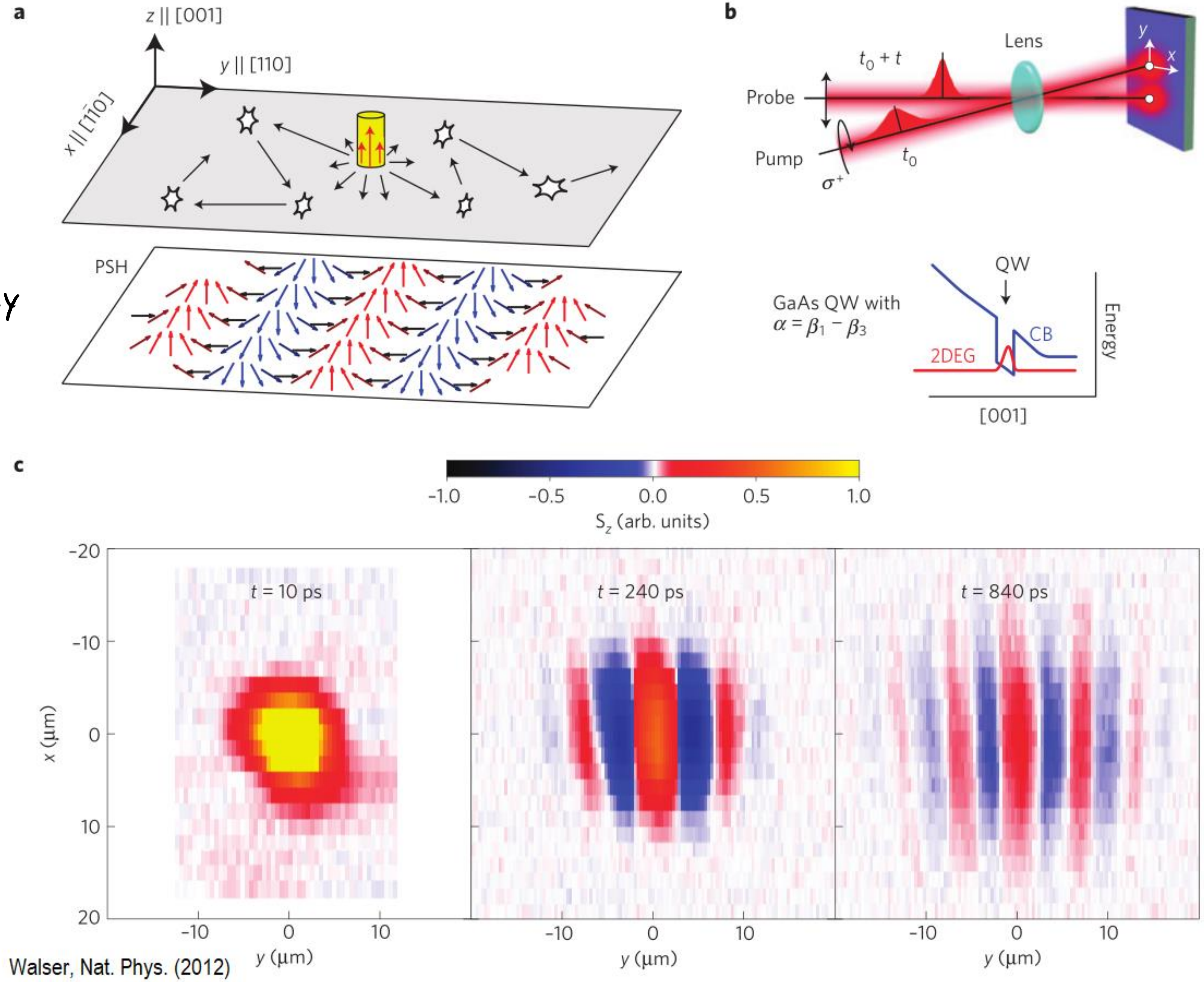
PERSISTANT SPIN HELIX

SPIN: SOI AND EFFECTIVE FIELD



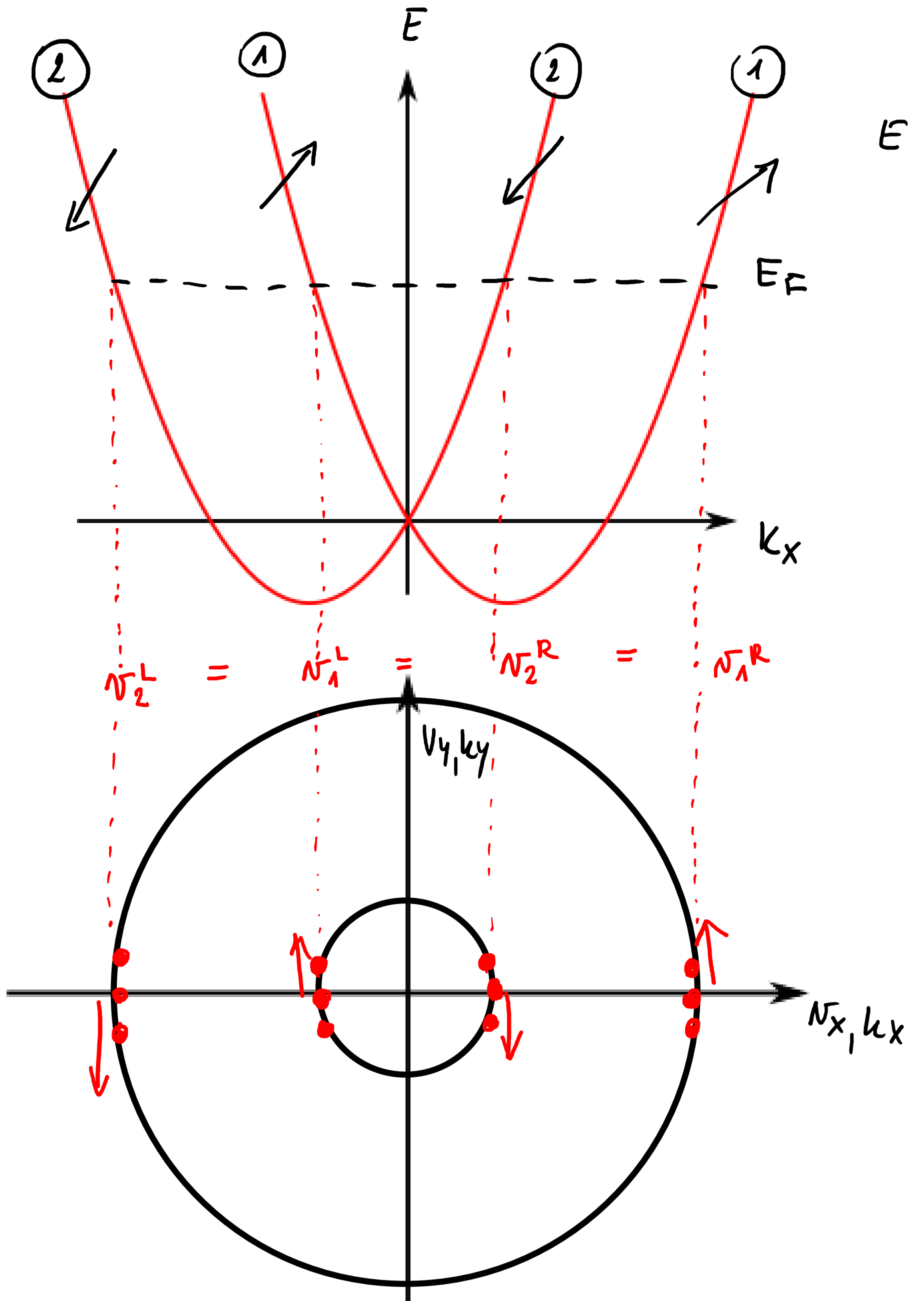
$y \sim k_y$

x

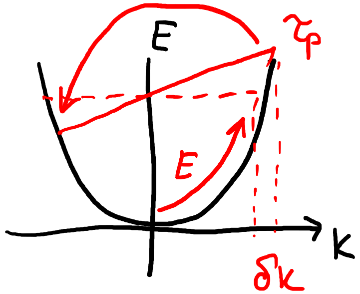


Walser, Nat. Phys. (2012)

SPIN: EDELSTEIN EFFECT



$E = 0$

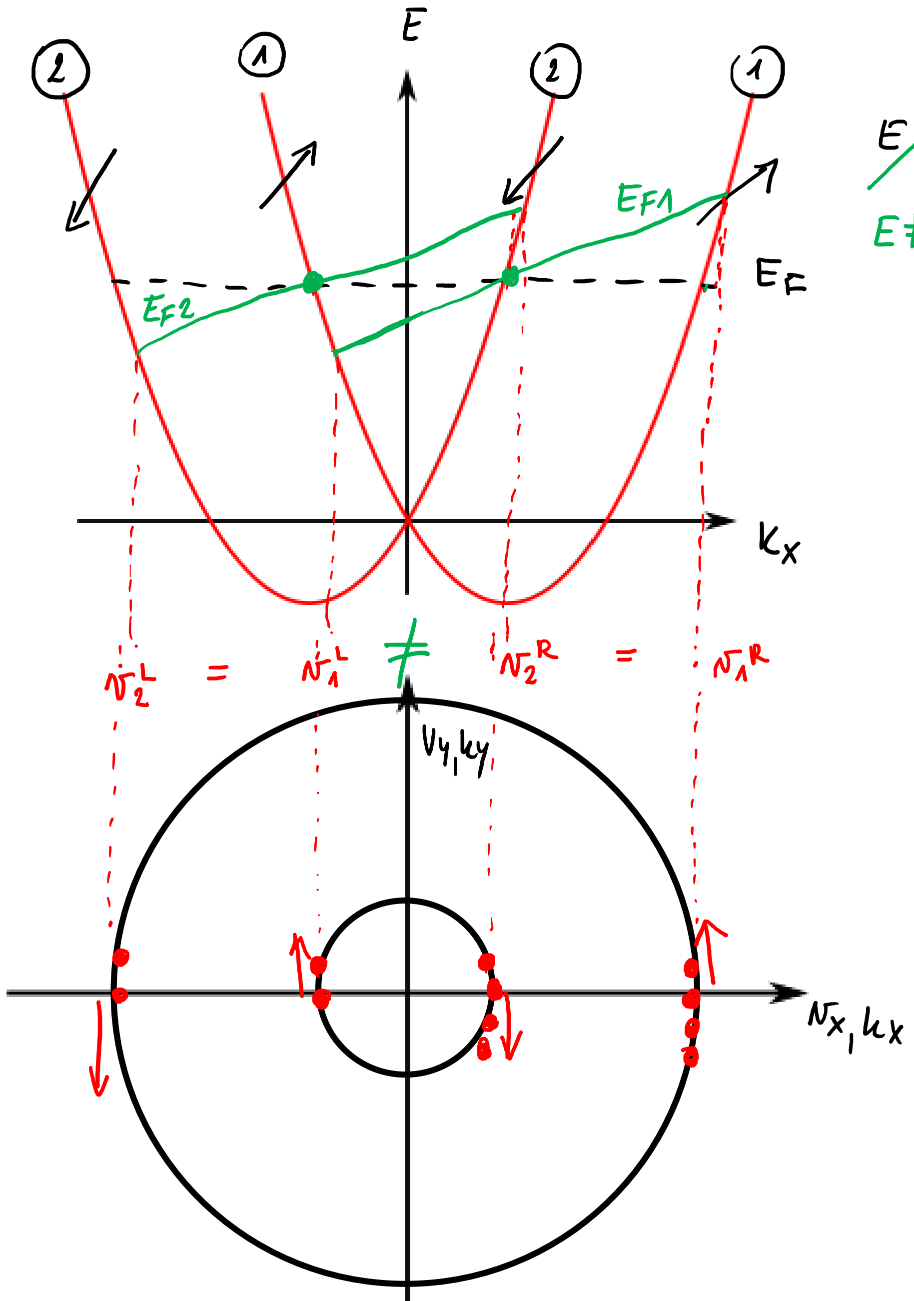


$\delta k = -eEt/\hbar$

$\delta k = -eE\tau_p/\hbar$

$$v_g = \frac{1}{\hbar}\frac{\partial E}{\partial k_x} \qquad E \propto \frac{\hbar^2 k^2}{2m} \pm \dots$$

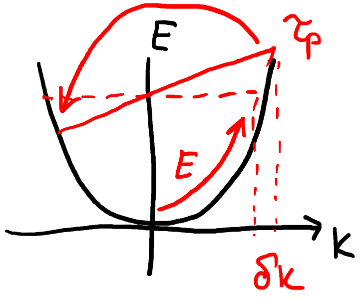
$\Rightarrow$ no net charge / spin current



~~E = 0~~

$E \neq 0$

$$v_g = \frac{1}{\hbar}\frac{\partial E}{\partial k_x}$$



$$E \propto \frac{\hbar^2 k^2}{2m} \pm \dots$$

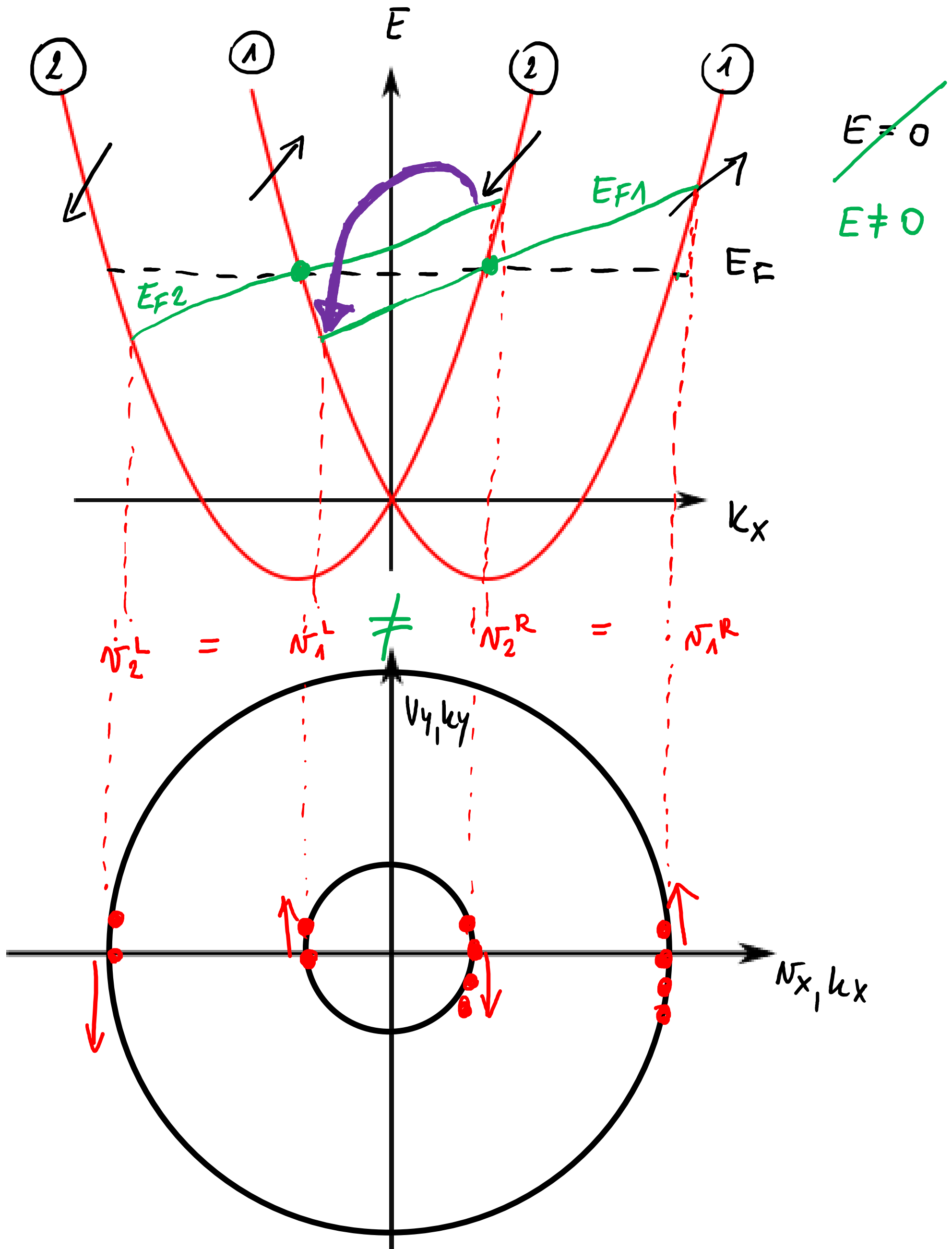
$$\delta k = -eEt/\hbar$$

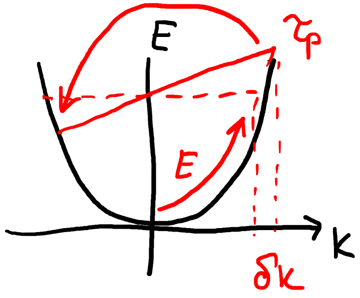
$$\delta k = -eE\tau_p/\hbar$$

⇒ net charge current

⇒ no net spin current

SPIN: EDELSTEIN EFFECT





$$\delta k = -eEt/\hbar$$
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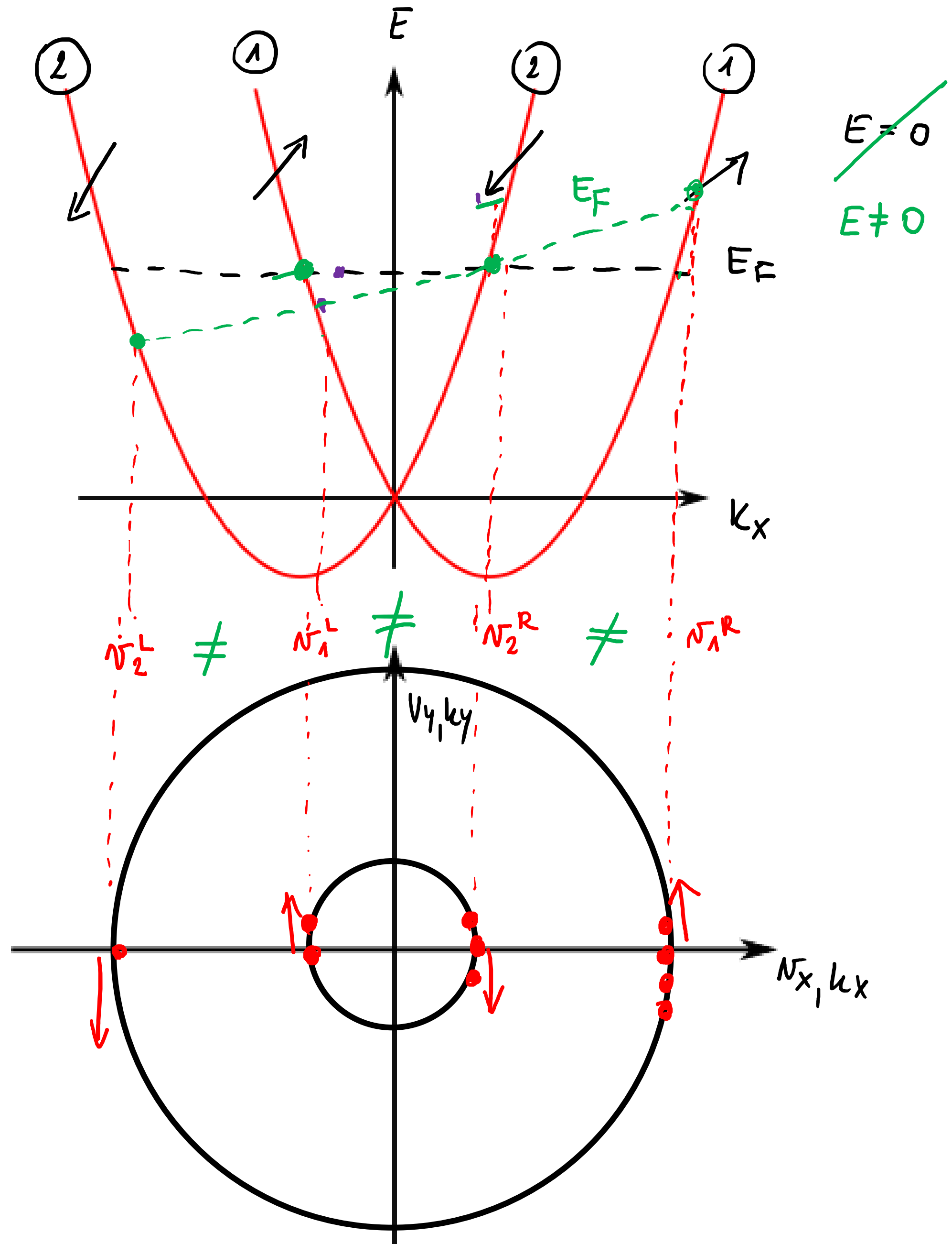
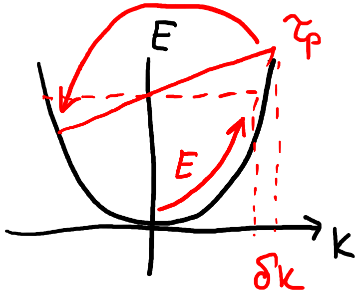
SPIN: EDELSTEIN EFFECT





$$\delta k = -eEt/\hbar$$

$$\delta k = -eE\tau_p/\hbar$$

$$v_g = \frac{1}{\hbar}\frac{\partial E}{\partial k_x} \qquad E \propto \frac{\hbar^2 k^2}{2m} \pm \dots$$

⇒ net charge current

⇒ net spin current

⇒ Send a current in a system w/ Rashba or other SOI and you will get spin current FOR FREE!

→ inverse is SPIN GALVANIC EFFECT