Twin-Width and Contraction Sequences - Set 2

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1 Warm up

Question 1. *Explicit versatile trees of contractions for the class of planar square grids.*

An *interval graph* is the intersection graph of a collection of intervals of the real line.

Question 2. *Is the class of interval graphs of bounded twin-width?*

2 Subdivisions

The s-subdivision of a graph G is the graph obtained from G, by replacing every edge of G by a path on $s+1$ edges.

Question 3. *Lower bound the twin-width of the s-subdivision of the n-vertex clique, in the regime $s = o(\log n)$.*

Question 4. *Upper bound the twin-width of the s-subdivision of the n-vertex clique, when $s \geqslant 2 \log n$.*

For Question 4, observe that $2 \log n$ upper bounds any leaf-to-leaf distance in a full binary tree with n leaves.

3 Contraction sequences via product structure

For the next question, it may paradoxically be easier to directly show that classes of bounded clique-width have bounded twin-width (and admit the known fact that classes of bounded treewidth have bounded clique-width).

Question 5. *Show that classes of bounded treewidth have bounded twin-width.*

The strong product $G_1 \boxtimes G_2$ of two graphs G_1, G_2 has vertex set $V(G_1) \times V(G_2)$ and an edge between two distinct vertices (u_1, u_2) and (v_1, v_2) of $V(G_1) \times V(G_2)$ whenever u_1, v_1 are equal or adjacent in G_1, and u_2, v_2 are equal or adjacent in G_2. For instance, the strong product of two paths is a grid with diagonals. A recent breakthrough in our understanding of planar graphs (and beyond) came from the following result.

Theorem 1 ([1])**.** *Every planar graph is a subgraph of the strong product of a path and a graph of treewidth at most 8.*

The upper bound of 8 has been decreased, but this is immaterial to the next question.

Question 6. *Use Question 5 and Theorem 1 to bound the twin-width of planar graphs.*

References

[1] V. Dujmovic, G. Joret, P. Micek, P. Morin, T. Ueckerdt, and D. R. Wood. Planar graphs have bounded queue-number. *J. ACM*, 67(4):22:1–22:38, 2020.