Problems, days 3-4:

1) In notation of Sidorenko's inequality, suppose the intersection of each chain C in P and chain C' in Q has size at most k. Prove that

$$e(P)\, e(Q) \ge n!/ (k!\, k^{n-k})$$

2) Let $G(P)$ be the comparability graph of poset P, that is, the vertices of $G(P)$ are the elements of the poset and two elements x and y are adjacent if either $x<y$ or $y<x$. Prove that $e(P)$ depends only on $G(P)$, i.e., that if the comparability graphs $G(P_1)$ and $G(P_2)$ of posets P_1 and P_2 are isomorphic, then $e(P_1)=e(P_2)$.

3) Describe vertices of the order and chain polytopes.

4) Let X be the set of n points in R^2 and let $Q=(X,<)$ be the poset defined in the lecture. Prove that there exists a permutation $\sigma \in S_n$ such that Q is isomorphic to P_σ