The Effective Shafarevich Conjecture

Algebra Colloquium, Charles University

Robin Visser
Faculty of Mathematics and Physics
Charles University

26 November 2024

Diophantine equations

Diophantine equations

Problem

Given a polynomial $f(x_1, \ldots, x_n) \in \mathbb{Z}[x_1, \ldots, x_n]$, can we understand the integer solutions to $f(x_1, \ldots, x_n) = 0$?

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  Are there other solutions?

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  $x = 569936821221962380720, y = -569936821113563493509, z = -472715493453327032$
  (Booker–Sutherland, 2019)

Diophantine equations

- Does there exist $x, y, z \in \mathbb{Z}$ such that $x^3 + y^3 + z^3 = 114$?

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$$a^2 + b^2 = d^2, \quad a^2 + c^2 = e^2, \quad b^2 + c^2 = f^2, \text{ and } a^2 + b^2 + c^2 = g^2 \text{ ?}$$

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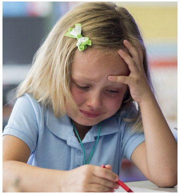
Diophantine equations are hard!



Hilbert's 10th Problem

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Hilbert's 10th Problem (1900)

Find an algorithm that decides whether any given polynomial equation $p(x_1, \ldots, x_n) = 0$ over the integers has an integral solution $x_1, \ldots, x_n \in \mathbb{Z}$.

David Hilbert

Hilbert's 10th Problem

Theorem (Matiyasevich–Robinson–Davis–Putnam, 1970)

No such algorithm exists!

Yuri Matiyasevich

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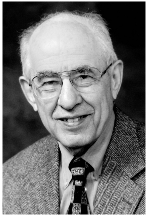

Hilary Putnam

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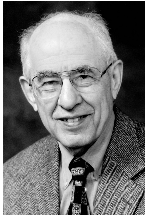

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- Okay, so trying to solve arbitrary Diophantine equations seems hopeless.

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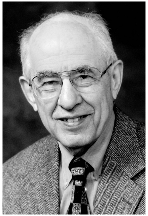

Hilary Putnam

- Okay, so trying to solve arbitrary Diophantine equations seems hopeless.
- Should we just pack up and leave?

Universal Diophantine system

Theorem (Jones, 1982)

There does not exist an algorithm which takes as input positive integers $x, v \in \mathbb{Z}_{>0}$ and outputs whether the Diophantine equation

$$\begin{aligned}
&(elg^2 + \alpha - (b - xy)q^2)^2 + (q - b^{5^{60}})^2 + (\lambda + q^4 - 1 - \lambda b^5)^2 + (\theta + 2z - b^5)^2 \\
&\quad + (u + t\theta - l)^2 + (y + m\theta - e)^2 + (n - q^{16})^2 + ((g + eq^3 + lq^5 \\
&\quad + (2(e - z\lambda)(1 + xb^5 + g)^4 + \lambda b^5 + \lambda b^5 q^4)q^4)(n^2 - n) + (q^3 - bl + l + \theta\lambda q^3 \\
&\quad + (b^5 - 2)q^5)(n^2 - 1) - r)^2 + (p - 2ws^2r^2n^2)^2 + (p^2k^2 - k^2 + 1 - \tau^2)^2 \\
&\quad + (4(c - ksn^2)^2 + \eta - k^2)^2 + (r + 1 + hp - h - k)^2 + (a - (wn^2 + 1)rsn^2)^2 \\
&\quad + (2r + 1 + \phi - c)^2 + (bw + ca - 2c + 4\alpha\gamma - 5\gamma - d)^2 + ((a^2 - 1)c^2 + 1 - d^2)^2 \\
&\quad + ((a^2 - 1)i^2c^4 + 1 - f^2)^2 + (((a + f^2(d^2 - a))^2 - 1)(2r + 1 + jc)^2 \\
&\quad + 1 - (d + of)^2)^2 + (((z + u + y)^2 + u)^2 + y - v)^2 = 0
\end{aligned}$$

has a solution over the positive integers.

Universal Diophantine system

When you learn that no algorithm exists to determine whether an arbitrary integral Diophantine equation has integer solutions.

Universal Diophantine system

Theorem (Jones, 1982)

There is no algorithm to test whether a polynomial $P(x_1, \ldots, x_9) \in \mathbb{Z}[x_1, \ldots, x_9]$ *has positive integer solutions* $x_1, \ldots, x_9$.

James Jones

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Conjecture (Baker 1968, Matiyasevich–Robinson 1975)

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- What about a polynomial $P(x_1, x_2)$ in two variables?

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Alan Baker

Plane curves

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Let's focus on plane algebraic curves (a single polynomial equation in two variables $f(x, y) = 0$).

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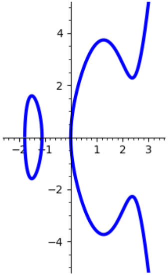
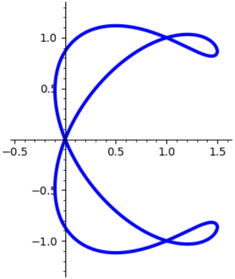
Genus 2 curve



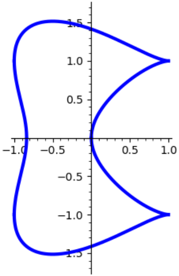
$$y^2 = x^5 - 2x^4 - 6x^3 + 8x^2 + 12x$$

Ampersand curve



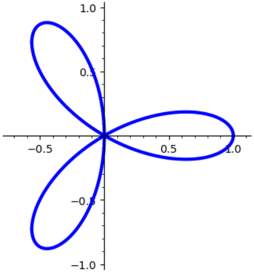
$$(y^2 - x^2)(x-1)(2x-3) = 4(x^2 + y^2 - 2x)^2$$

Bicuspid curve



$$(x^2 - 1)(x-1)^2 = -(y^2 - 1)^2$$

Three leaf clover



$$x^4 + 2x^2y^2 + y^4 - x^3 + 3xy^2 = 0$$

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We can classify curves according to their **genus**.

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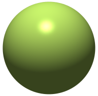
- **Genus 0 curves** (spherical geometry)



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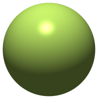

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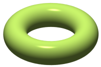

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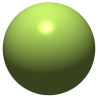

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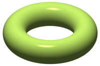

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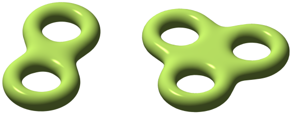

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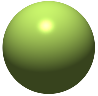

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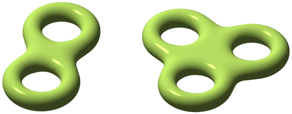

Theorem (Genus–degree formula)

The genus g of a smooth irreducible degree d plane curve $C/\mathbb{Q}$ is $g = \frac{(d-1)(d-2)}{2}$.

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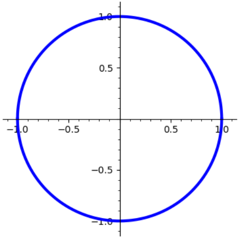
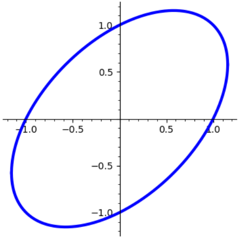
Circle



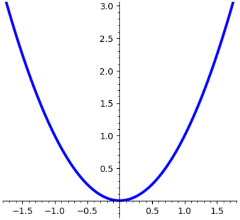
$x^2 + y^2 = 1$

Ellipse



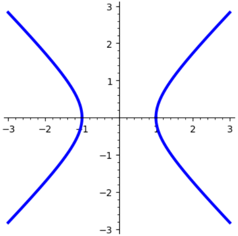
$x^2 - xy + y^2 = 1$

Parabola



$y = x^2$

Hyperbola



$y^2 + 1 = x^2$

Genus 0 curves

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$C(\mathbb{Q})$

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$$\begin{array}{c} C(\mathbb{R}) \\ \Big\uparrow \\ C(\mathbb{Q}) \end{array}$$

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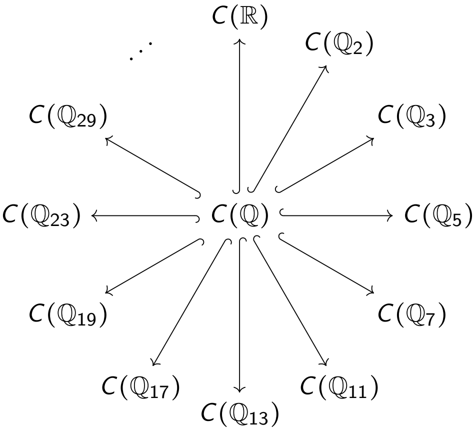

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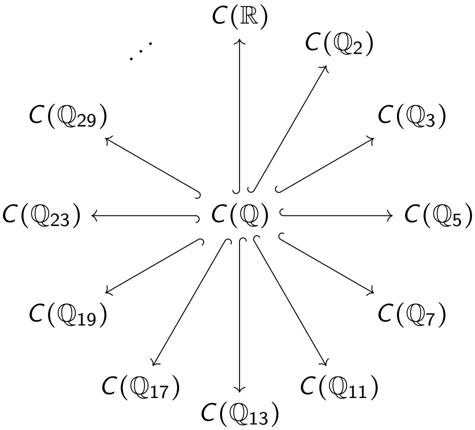
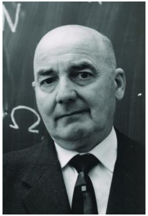


Theorem (Hasse-Minkowski)

Let $C/\mathbb{Q}$ be a smooth genus 0 curve. Then $C(\mathbb{Q}) \neq \varnothing$ if and only if $C(\mathbb{R}) \neq \varnothing$ and $C(\mathbb{Q}_p) \neq \varnothing$ for all primes p .



Helmut Hasse

Hermann Minkowski

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Find all $x, y \in \mathbb{Q}$ such that $x^2 + y^2 = 1$.

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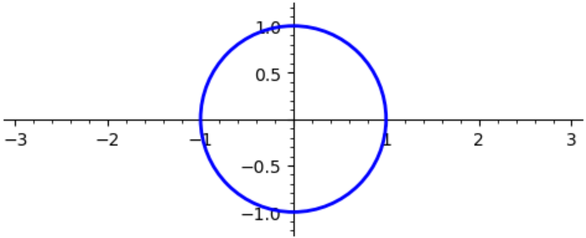

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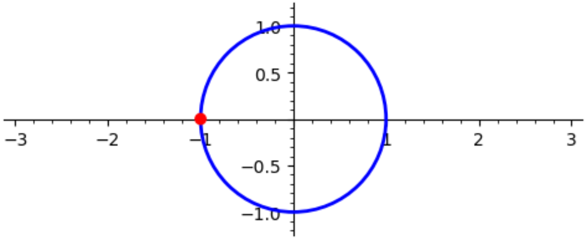

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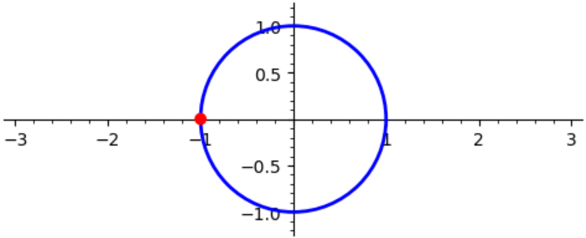

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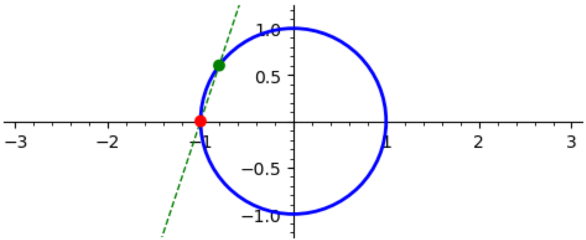

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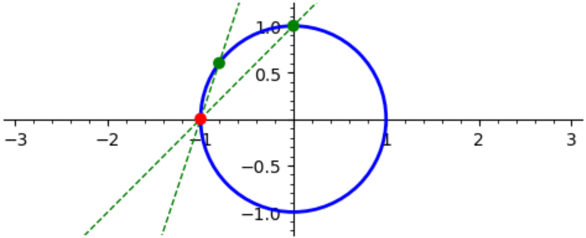

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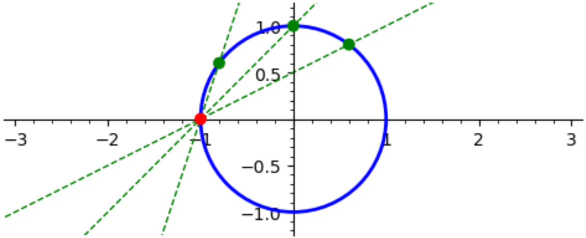

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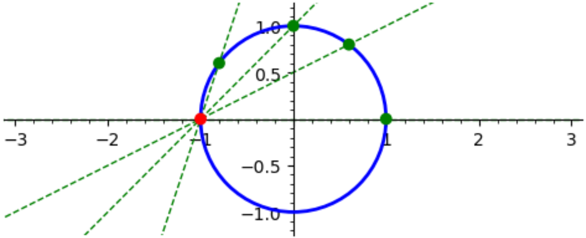

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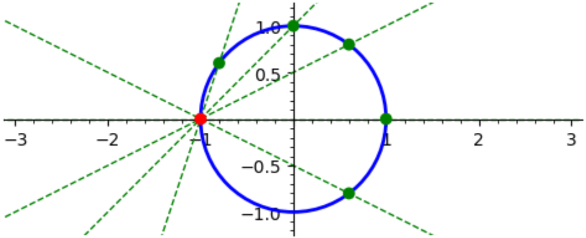

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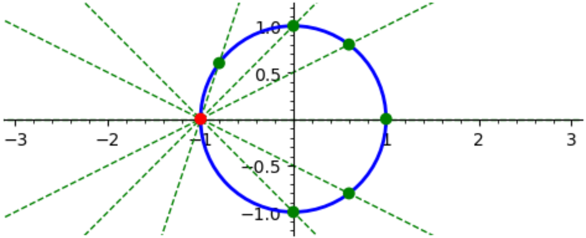

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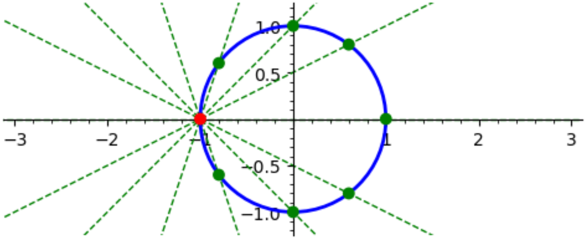

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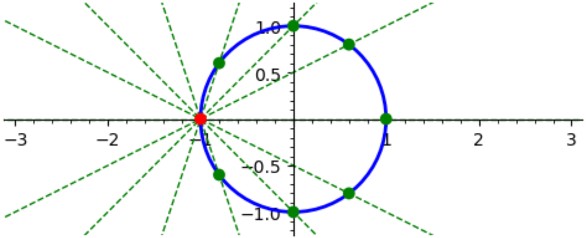

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- **Solution:** $(x, y) = (-1, 0)$ or $\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}\right)$ for some $t \in \mathbb{Q}$.

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- In this case, $C/\mathbb{Q}$ is an **elliptic curve**. These are precisely the dimension 1 **abelian varieties**!

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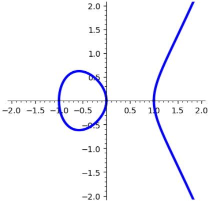

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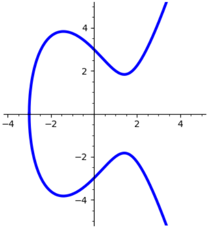

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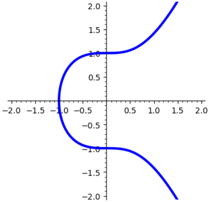

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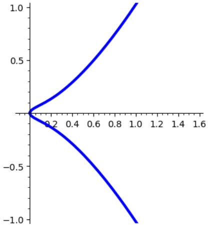

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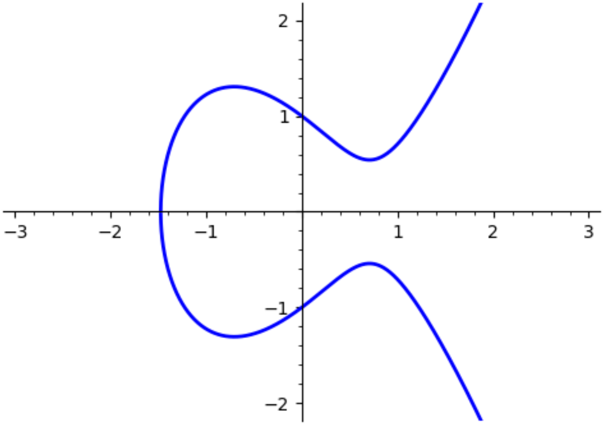

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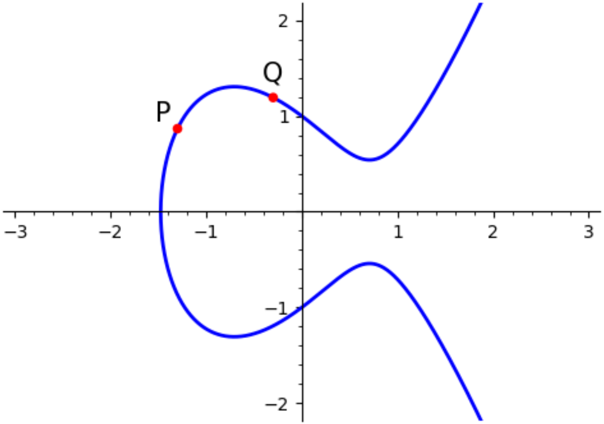

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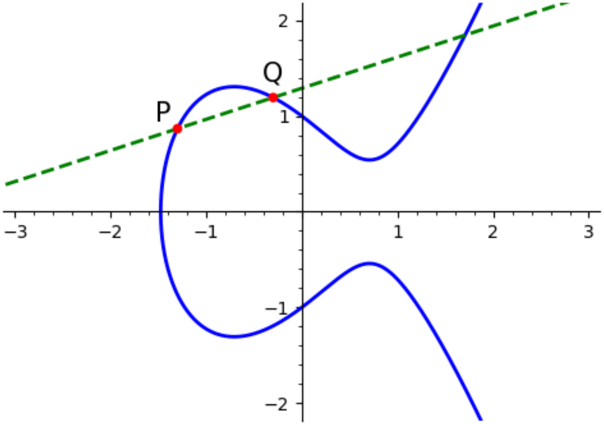

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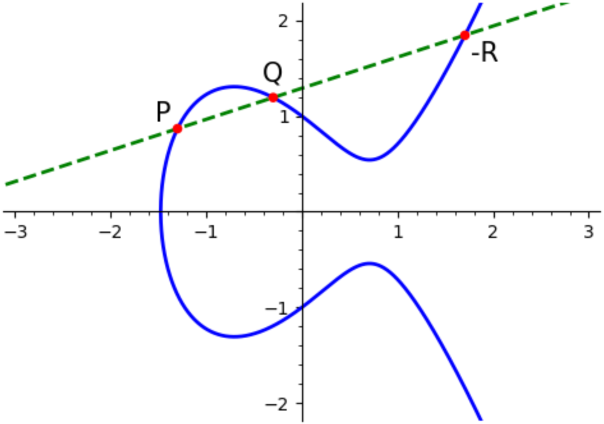

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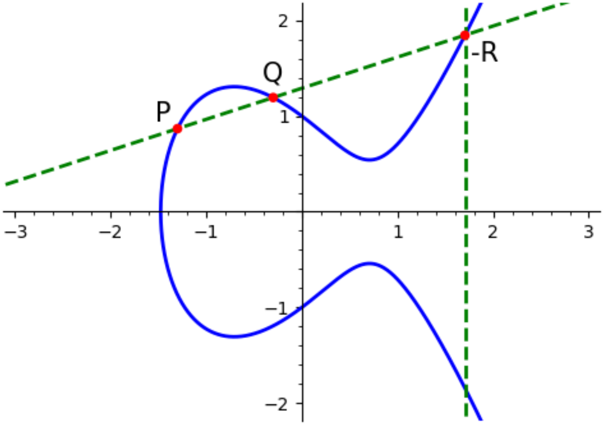

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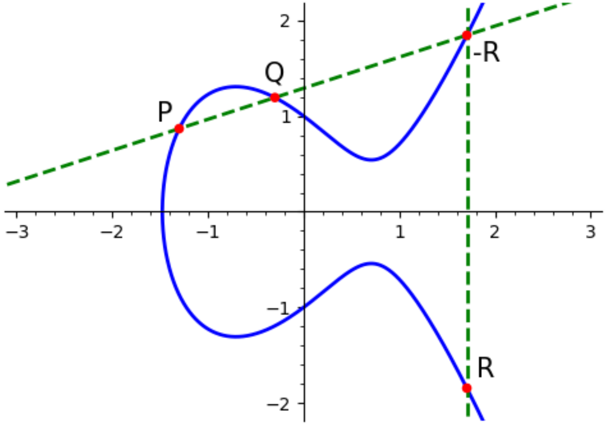

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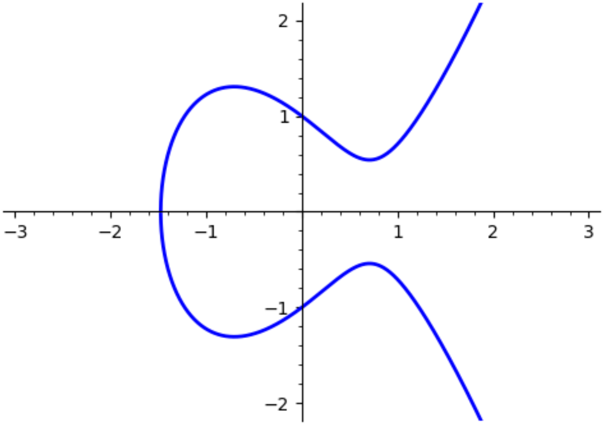

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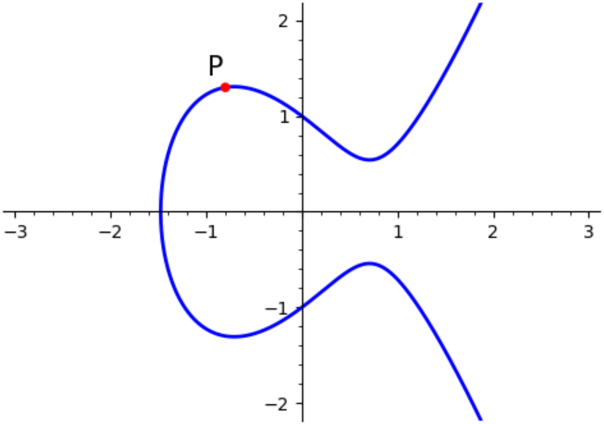

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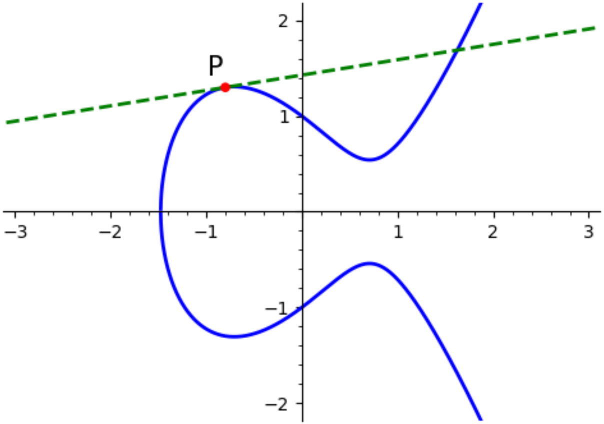

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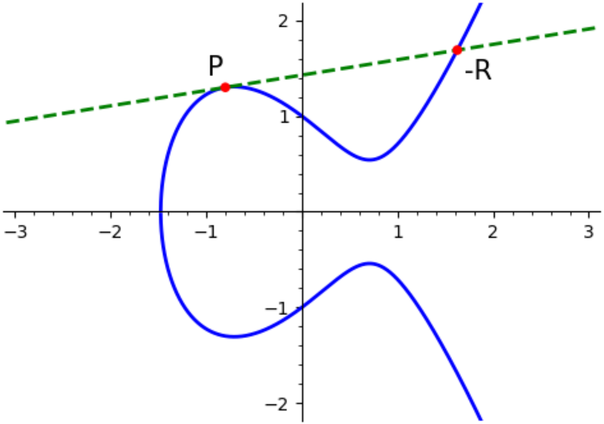

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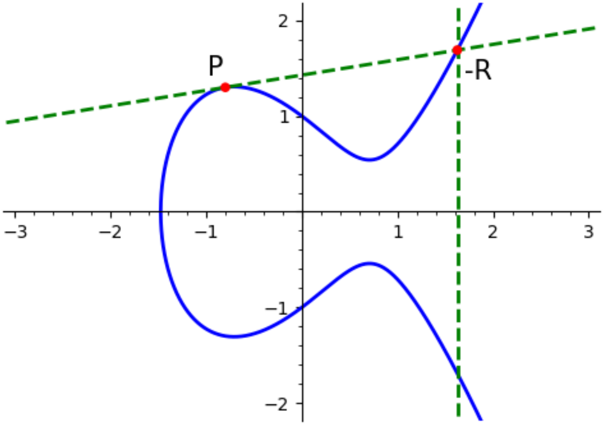

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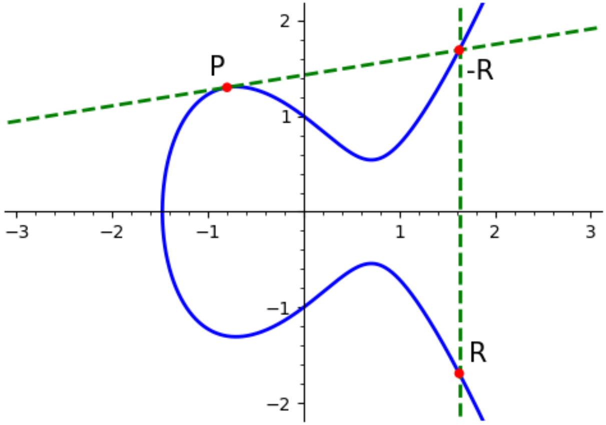

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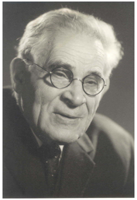

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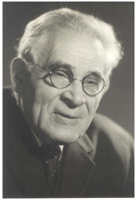

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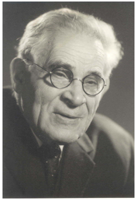

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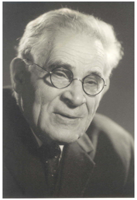

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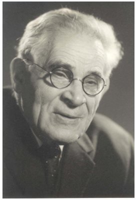

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Genus ≥ 2 curves

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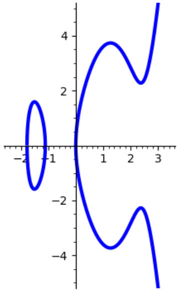
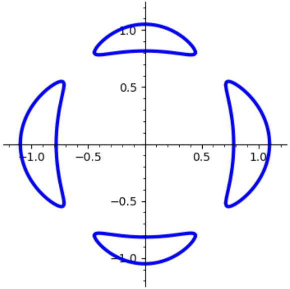
Now it becomes much harder to compute all rational points!

Genus 2 hyperelliptic



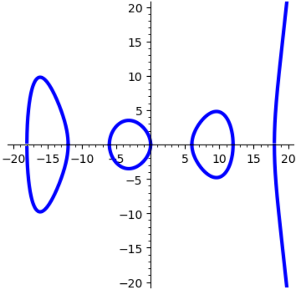
$$y^2 = x^5 - 2x^4$$
$$-6x^3 + 8x^2 + 12x$$

Genus 3 plane quartic



$$144(x^4 + y^4) - 225(x^2 + y^2)$$
$$+350x^2y^2 + 81 = 0$$

Genus 3 hyperelliptic



$$y^2 = x^7 - 14x^5$$
$$+49x^3 - 36x$$

Genus ≥ 2 curves

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Find all $x, y \in \mathbb{Q}$ such that $y^2 = x^6 + x^2 + 1$.

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(Balakrishnan–Dogra–Müller–Tuitman–Vonk 2019)

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- Known points: $(0, \pm 8)$. Are there any others??

Mordell and Shafarevich

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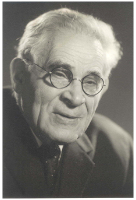
Conjecture (Mordell 1922)

Any smooth curve $C/\mathbb{Q}$ of genus at least 2 has only finitely many rational points.



Louis J. Mordell

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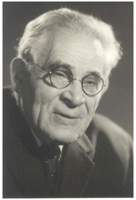

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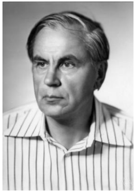

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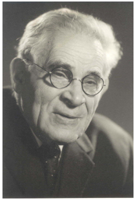

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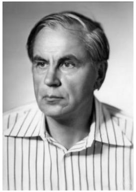

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Finally, in 1983, Faltings proved the Shafarevich conjecture, thus proving the Mordell conjecture!

Theorem (Faltings 1983)

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Aleksei N. Parshin

Gerd Faltings

Faltings's theorem

Faltings after proving the Mordell conjecture

Faltings's theorem

Figure: Leslie Valiant (Nevanlinna Prize), Michael Freedman, Gerd Faltings, Simon Donaldson (Fields Medalists), 1986, Berkeley ICM.

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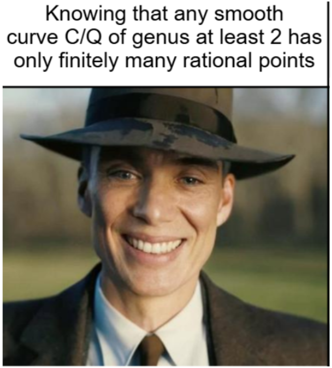

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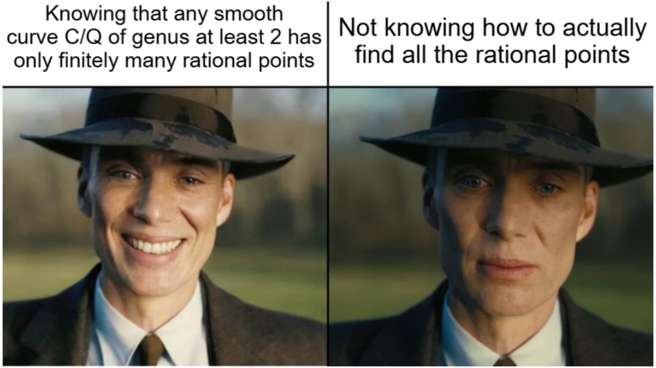

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In practice, there are many approaches one could try:

- Local methods
- Quotients
- Descent
- Mordell-Weil sieve
- Chabauty-Coleman (see also Kim's non-abelian Chabauty, quadratic Chabauty)

Effective Shafarevich

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There exists an algorithm which can explicitly compute $C(\mathbb{Q})$ given any smooth curve $C/\mathbb{Q}$ of genus at least 2.

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There exists an algorithm which can explicitly compute all smooth genus g curves C/K with good reduction outside S, for any number field K, integer integer $g \geq 2$, and finite set of primes S of K.

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Even the case $d = 2$, $K = \mathbb{Q}$, $S = \{2\}$ is still an open problem!

Elliptic Curves Summary

- $E(S)$ denotes the set of elliptic curves $E/\mathbb{Q}$ with good reduction outside S.

Set S	$\lvert E(S)\rvert$	**Authors**	**Year**
$\varnothing$	0	Tate (proof published by Ogg)	1965
$\{2\}$	24	Ogg	1965
$\{2,3\}$	752	Coghlan, Stephens	1967, 1965
$\{11\}$	12	Agrawal–Coates–Hunt–Van der Poorten	1980
$\{2,p\},\ p \in \{5,\ldots,23\}$	$280, 288, \ldots$	Cremona–Lingham	2007
$\{2,3,23\}$	5520	Koutsianas	2015
$\{2,3,5,7,11\}$	592 192	von Känel–Matschke	2016
$\{2,3,5,7,11,13\}$	4 576 128	Best–Matschke	2020
$\{2,3,5,7,\ldots,23\}$	1 390 818 304*	Matschke	2021

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Cases 2 and 3 can easily be done. Case 1 is hard!

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List all genus 2 curves $C/\mathbb{Q}$ whose Jacobians have good reduction away from 2.

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- $C : y^2 = x^5 + 28x^4 - 868x^3 - 6160x^2 + 43076x - 149072$ has bad reduction at $\{2, 3, 11\}$.

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Can we actually find all such curves?

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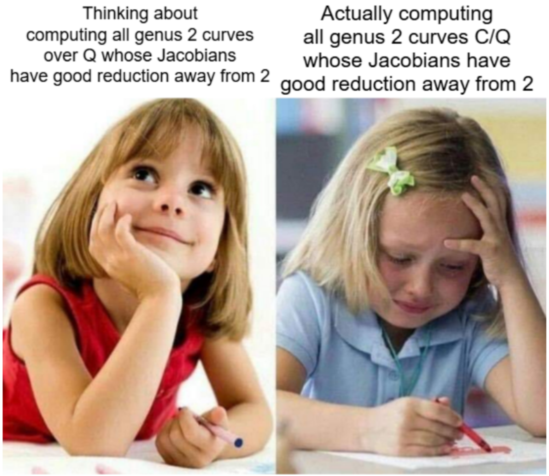

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- Compute all possible 2-torsion fields $\mathbb{Q}(J[2])$.
- Solve the T-unit equations $x+y=1$ for $x, y \in O_T^\times$ over $\mathbb{Q}(J[2])$ where T is the primes in $\mathbb{Q}(J[2])$ lying above S.

Further optimisations

Let $\psi_1, \psi_2, \ldots, \psi_t$ be a set of T-unit generators over $\mathbb{Q}(J[2])$. Let $a_{k,i,j} \in \mathbb{Z}$ be given by

$$\alpha_i - \alpha_j = \psi_1^{a_{1,i,j}} \psi_2^{a_{2,i,j}} \cdots \psi_t^{a_{t,i,j}}$$

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$$\alpha_i - \alpha_j = \psi_1^{a_{1,i,j}} \psi_2^{a_{2,i,j}} \cdots \psi_t^{a_{t,i,j}}$$

Constraints on $a_{k,i,j}$:

- Galois constraints: For all $\sigma \in \mathrm{Gal}(\mathbb{Q}(J[2])/\mathbb{Q})$, $a_{f_\sigma(k),g_\sigma(i),g_\sigma(j)} = a_{k,i,j}$.
- Cluster pictures (using that J has good reduction at odd primes).
- Solving simple T-unit equations (i.e. $\tau + \sigma(\tau) = 1$ for some $\sigma \in \mathrm{Gal}(\mathbb{Q}(J[2])/\mathbb{Q})$).

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Solving the linear system:

- Brute force
- Closest vector problem
- Integer programming

List of 512 genus 2 curves $C/\mathbb{Q}$

Using these methods, we tabulate the 512 known genus 2 curves $C/\mathbb{Q}$ whose Jacobian has good reduction away from 2:

Label	Simplified Weierstrass model $y^2 = f(x)$	$\Delta_{\min}$	N	Rank
1	$(x-1)(x+1)(x^2-2x-1)(x^2+1)$	-2^9	2^8	0
2	$-(x-1)x(x+1)(x^2-2)$	2^{15}	2^{13}	0
3	$-x(x^2+1)(x^2+2)$	2^{15}	2^{14}	0
4	$(x-2)(x+2)(x^4-4x^2-4)$	-2^{16}	2^{12}	1
5	$(x^2+4)(x^4+4x^2-4)$	2^{16}	2^{12}	0
6	$(x-1)x(x+1)(x^2+1)$	-2^{16}	2^{12}	0
7	$x(x^4+1)$	2^{16}	2^{16}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

8	$-(x-1)x(x+1)(x^2-2x-1)$	2^{17}	2^{13}	0
9	$x(x+4)(x^4-12x^2+16x-4)$	2^{17}	2^{13}	0
10	$-x(x^4+4x^3+4x^2+1)$	2^{17}	2^{17}	0
11	$x(x^4+4x^3+4x^2+1)$	2^{17}	2^{17}	0
12	$-(x+1)(x^4+1)$	2^{18}	2^{17}	0
13	$-(x-1)(x^4+1)$	2^{18}	2^{17}	2
14	$x(x^4+2x^2-1)$	-2^{18}	2^{18}	1
15	$(x-2)(x+2)(x^4-4x^2+8)$	2^{19}	2^{12}	0
16	$(x^2+4)(x^4+4x^2+8)$	-2^{19}	2^{12}	0
17	$x(x^2-2x-1)(x^2+1)$	-2^{19}	2^{12}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

18	$(x-1)(x+1)(x^4-2x^2+2)$	2^{19}	2^{13}	1
19	$(x^2+1)(x^4+2x^2+2)$	-2^{19}	2^{13}	1
20	$-(x^2+1)(x^4+2x^2+2)$	-2^{19}	2^{13}	0
21	$-(x-1)(x+1)(x^4-2x^2+2)$	2^{19}	2^{13}	0
22	$-x(x^4+2x^2+2)$	2^{19}	2^{18}	1
23	$-x(x^4-2x^2+2)$	2^{19}	2^{18}	1
24	$x(x^4+4x^3+10x^2+8x+2)$	2^{19}	2^{18}	1
25	$-x(x^4+4x^3+10x^2+8x+2)$	2^{19}	2^{18}	1
26	$(x+1)(x^4-2)$	-2^{19}	2^{19}	0
27	$(x-1)(x^4-2)$	-2^{19}	2^{19}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

28	$x(x^4+4x^3+2x^2-4x-1)$	2^{19}	2^{19}	0
29	$-x(x^4+4x^3+2x^2-4x-1)$	2^{19}	2^{19}	1
30	$-(x-1)(x^4+2x^2-1)$	-2^{20}	2^{19}	1
31	$(x-1)(x^4+2x^2-1)$	-2^{20}	2^{19}	2
32	$(x-1)(x^4-2x^2-1)$	-2^{20}	2^{19}	2
33	$-(x-1)(x^4-2x^2-1)$	-2^{20}	2^{19}	0
34	$-(x^2-2)(x^4-2x^2-1)$	-2^{21}	2^{12}	0
35	$(x^2+2)(x^4+2x^2-1)$	2^{21}	2^{12}	0
36	$-(x^2+2)(x^4+2x^2-1)$	2^{21}	2^{12}	0
37	$(x^2-2)(x^4-2x^2-1)$	-2^{21}	2^{12}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

38	$(x^2+6x+1)(x^4+4x^3-2x^2+4x+1)$	-2^{21}	2^{13}	1
39	$-(3x^2-2x+3)(x^4+4x^3-2x^2+4x+1)$	2^{21}	2^{13}	1
40	$(x^2+4)(x^4-8)$	2^{21}	2^{15}	2
41	$(x-2)(x+2)(x^4-8x^2+8)$	2^{21}	2^{15}	1
42	$(x^2+4)(x^4+8x^2+8)$	-2^{21}	2^{15}	1
43	$(x-2)(x+2)(x^4-8)$	-2^{21}	2^{15}	1
44	$(x^2+1)(x^4-2)$	2^{21}	2^{15}	1
45	$-(x^2+1)(x^4+4x^2+2)$	-2^{21}	2^{15}	1
46	$(x-1)(x+1)(x^4-4x^2+2)$	2^{21}	2^{15}	1
47	$-(x-1)(x+1)(x^4-2)$	-2^{21}	2^{15}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

48	$(x-1)(x+1)(x^4-2)$	-2^{21}	2^{15}	1
49	$(x^2+1)(x^4+4x^2+2)$	-2^{21}	2^{15}	1
50	$-(x-1)(x+1)(x^4-4x^2+2)$	2^{21}	2^{15}	1
51	$-(x^2+1)(x^4-2)$	2^{21}	2^{15}	0
52	$-x(x^4+2)$	2^{21}	2^{20}	1
53	$x(x^4-2)$	-2^{21}	2^{20}	1
54	$-x(x^4+4x^2+2)$	2^{21}	2^{20}	0
55	$-x(x^4-4x^2+2)$	2^{21}	2^{20}	2
56	$(x-1)(2x^4-1)$	-2^{21}	2^{20}	1
57	$-(x+1)(x^4-4x^3+2x^2+4x-1)$	2^{21}	2^{20}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

58	$x(2x^4+8x^3+8x^2-1)$	2^{21}	2^{20}	1
59	$-(x+1)(x^4-4x^3-2x^2+4x-1)$	2^{21}	2^{20}	2
60	$-(x-1)(x^4+4x^3-2x^2-4x-1)$	2^{21}	2^{20}	1
61	$(x+1)(2x^4-1)$	-2^{21}	2^{20}	0
62	$-(x-1)(x^4+4x^3+2x^2-4x-1)$	2^{21}	2^{20}	1
63	$-x(2x^4+8x^3+8x^2-1)$	2^{21}	2^{20}	2
64	$-(x^2+2)(2x^4+4x^2+1)$	-2^{22}	2^{13}	1
65	$(x^2-2)(2x^4-4x^2+1)$	2^{22}	2^{13}	1
66	$-(x^2-2)(2x^4-4x^2+1)$	2^{22}	2^{13}	0
67	$(x^2+2)(2x^4+4x^2+1)$	-2^{22}	2^{13}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

68	$(x^2+1)(x^4+1)$	-2^{22}	2^{14}	2
69	$-x(x^2-2x-1)(x^2+2x-1)$	2^{22}	2^{14}	1
70	$(x-1)(x+1)(x^4+1)$	2^{22}	2^{14}	1
71	$-x(x^4+6x^2+1)$	2^{22}	2^{14}	0
72	$-(x^2+1)(x^4+1)$	-2^{22}	2^{14}	0
73	$(x^2+8)(x^4+8x^2+8)$	-2^{22}	2^{14}	1
74	$(x^2-8)(x^4-8x^2+8)$	2^{22}	2^{14}	0
75	$(x+1)(x^2-2x-1)(x^2+1)$	-2^{23}	2^{15}	0
76	$x(x^2-2x+2)(x^2+2)$	2^{23}	2^{15}	0
77	$(x-1)(x^2+1)(x^2+2x-1)$	-2^{23}	2^{15}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

78	$-x(x^2-2x+2)(x^2+2)$	2^{23}	2^{15}	0
79	$-(x-1)(x^2-2x-1)(x^2+1)$	-2^{23}	2^{15}	0
80	$-(x+1)(x^2+1)(x^2+2x-1)$	-2^{23}	2^{15}	1
81	$-(x^2-2)(x^4-2x^2+2)$	2^{24}	2^{12}	1
82	$(x^2+2)(x^4+2x^2+2)$	-2^{24}	2^{12}	1
83	$-(x^2+2)(x^4+2x^2+2)$	-2^{24}	2^{12}	0
84	$(x^2-2)(x^4-2x^2+2)$	2^{24}	2^{12}	0
85	$-(x^2+1)(x^4-2x^2-1)$	2^{24}	2^{16}	2
86	$(x-1)(x+1)(x^4+2x^2-1)$	-2^{24}	2^{16}	2
87	$x(x^4-4x^3-2x^2-4x+1)$	-2^{24}	2^{16}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

88	$-(x-1)(x+1)(x^4+2x^2-1)$	-2^{24}	2^{16}	1
89	$(x^2+1)(x^4-2x^2-1)$	2^{24}	2^{16}	1
90	$x(x^4+4x^3-2x^2+4x+1)$	-2^{24}	2^{16}	0
91	$-(x-1)(x^4+4x^2-4)$	-2^{24}	2^{16}	0
92	$(x-1)(x^4+4x^2-4)$	-2^{24}	2^{16}	1
93	$x(x^4-8x^3+18x^2+8x+1)$	2^{25}	2^{13}	0
94	$-2x(x^2+1)(x^2+2)$	2^{25}	2^{13}	0
95	$-2(x-1)x(x+1)(x^2-2)$	2^{25}	2^{14}	0
96	$x(x^4-4x^3-6x^2+4x+1)$	2^{25}	2^{17}	1
97	$-x(x^4-4x^3+10x^2+4x+1)$	2^{25}	2^{17}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

98	$-x(x^4+4x^3-2x^2-12x+1)$	2^{25}	2^{17}	0
99	$x(x^4+4x^3-2x^2-12x+1)$	2^{25}	2^{17}	0
100	$-x(x+2)(x^4-4x^2+2)$	2^{25}	2^{17}	0
101	$-(x^2+1)(x^4+4x^3+6x^2+4x+3)$	-2^{25}	2^{17}	0
102	$x(x+2)(x^4-4x^2+2)$	2^{25}	2^{17}	2
103	$(x^2+1)(x^4+4x^3+6x^2+4x+3)$	-2^{25}	2^{17}	2
104	$(x^2+1)(x^4-4x^2-4)$	2^{26}	2^{12}	1
105	$-(x-1)(x+1)(x^4+4x^2-4)$	-2^{26}	2^{12}	0
106	$-x(x^2-2x+2)(x^2+2x+2)$	2^{26}	2^{14}	0
107	$(x^2-2)(x^4-2)$	-2^{26}	2^{15}	2

List of 512 genus 2 curves $C/\mathbb{Q}$

108	$(x^2-2)(x^4-4x^2+2)$	2^{26}	2^{15}	2
109	$(x^2+2)(x^4+4x^2+2)$	-2^{26}	2^{15}	2
110	$(x^2+2)(x^4-2)$	2^{26}	2^{15}	1
111	$-(x^2+2)(x^4-2)$	2^{26}	2^{15}	1
112	$-(x^2+2)(x^4+4x^2+2)$	-2^{26}	2^{15}	1
113	$-(x^2-2)(x^4-4x^2+2)$	2^{26}	2^{15}	1
114	$-(x^2-2)(x^4-2)$	-2^{26}	2^{15}	0
115	$-2x(x^4+1)$	2^{26}	2^{16}	1
116	$x(x^2-2)(x^2+2)$	-2^{26}	2^{16}	0
117	$(x-1)(x^2-2x-1)(x^2+2x-1)$	2^{26}	2^{16}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

118	$-(x-1)(x^2-2x-1)(x^2+2x-1)$	2^{26}	2^{16}	1
119	$(x-1)(x+1)(x^4-8x^2+8)$	2^{27}	2^{13}	1
120	$(x^2+1)(x^4+8x^2+8)$	-2^{27}	2^{13}	1
121	$-(x^2+1)(x^4+8x^2+8)$	-2^{27}	2^{13}	0
122	$-(x-1)(x+1)(x^4-8x^2+8)$	2^{27}	2^{13}	0
123	$-(2x-3)(x^4+4x^3-6x^2-4x+1)$	2^{27}	2^{13}	0
124	$-(x^2+4)(x^4+4x^2+2)$	-2^{27}	2^{14}	1
125	$(x-2)(x+2)(x^4-4x^2+2)$	2^{27}	2^{14}	1
126	$(x^2+4)(x^4+4x^2+2)$	-2^{27}	2^{14}	0
127	$-(x-2)(x+2)(x^4-4x^2+2)$	2^{27}	2^{14}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

128	$2x(x^4+4x^3+4x^2+1)$	2^{27}	2^{17}	0
129	$-2x(x^4+4x^3+4x^2+1)$	2^{27}	2^{17}	2
130	$x(x^2-4x+2)(x^2+2)$	-2^{28}	2^{16}	1
131	$-x(x^2-4x+2)(x^2+2)$	-2^{28}	2^{16}	0
132	$-2(x-1)(x^4+1)$	2^{28}	2^{17}	0
133	$2(x-1)(x^4+1)$	2^{28}	2^{17}	0
134	$-x(x^4+4x^2-4)$	-2^{28}	2^{18}	2
135	$-x(x^4-4x^2-4)$	-2^{28}	2^{18}	0
136	$2x(x^4+2x^2-1)$	-2^{28}	2^{18}	1
137	$(x-1)(x^4+4x^3+2x^2-4x-7)$	-2^{28}	2^{18}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

138	$-(x-1)(x^4+4x^3+2x^2-4x-7)$	-2^{28}	2^{18}	1
139	$x(x+1)(x^4+4x^2-4)$	-2^{28}	2^{18}	1
140	$-(x-1)x(x^4+4x^2-4)$	-2^{28}	2^{18}	1
141	$-2(x^2+1)(x^4+2x^2+2)$	-2^{29}	2^{12}	0
142	$2(x-1)(x+1)(x^4-2x^2+2)$	2^{29}	2^{12}	0
143	$2x(x^2+1)(x^2+2x-1)$	-2^{29}	2^{14}	0
144	$2x(x^2-2x-1)(x^2+1)$	-2^{29}	2^{14}	0
145	$(x-1)(x^4+8x^3+4x^2-16x+4)$	2^{29}	2^{15}	0
146	$-(x-1)(x^4+8x^3+4x^2-16x+4)$	2^{29}	2^{15}	1
147	$-x(x^4+4x^3+4x^2-8x+4)$	2^{29}	2^{17}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

148	$x(x^4+4x^3+4x^2-8x+4)$	2^{29}	2^{17}	1
149	$-x(x^4-4x^3+12x^2-8x+4)$	2^{29}	2^{17}	0
150	$x(x^4-4x^3+12x^2-8x+4)$	2^{29}	2^{17}	0
151	$-2x(x^4-2x^2+2)$	2^{29}	2^{18}	1
152	$-2x(x^4+2x^2+2)$	2^{29}	2^{18}	1
153	$2x(x^4+4x^3+10x^2+8x+2)$	2^{29}	2^{18}	1
154	$-2x(x^4+4x^3+10x^2+8x+2)$	2^{29}	2^{18}	1
155	$-2x(x^4+4x^3+2x^2-4x-1)$	2^{29}	2^{19}	1
156	$2x(x^4+4x^3+2x^2-4x-1)$	2^{29}	2^{19}	2
157	$-2(x+1)(x^4-2)$	-2^{29}	2^{19}	2

List of 512 genus 2 curves $C/\mathbb{Q}$

158	$2(x+1)(x^4-2)$	-2^{29}	2^{19}	0
159	$-(x+4)(x^4-12x^2+16x-4)$	2^{29}	2^{19}	1
160	$-(x+1)(x^4-4x^3-6x^2+4x+1)$	2^{29}	2^{19}	0
161	$-(x+2)(4x^4-12x^2+8x-1)$	2^{29}	2^{19}	1
162	$(x+2)(4x^4-12x^2+8x-1)$	2^{29}	2^{19}	0
163	$-(x-1)(x^4+4x^3-6x^2-4x+1)$	2^{29}	2^{19}	1
164	$(x+4)(x^4-12x^2+16x-4)$	2^{29}	2^{19}	0
165	$(x+1)(x^4+4x^3-6x^2-4x+1)$	2^{29}	2^{19}	2
166	$-(x+1)(x^4+4x^3-6x^2-4x+1)$	2^{29}	2^{19}	1
167	$-2(x-1)(x^4-2x^2-1)$	-2^{30}	2^{19}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

168	$2(x-1)(x^4-2x^2-1)$	-2^{30}	2^{19}	0
169	$2(x-1)(x^4+2x^2-1)$	-2^{30}	2^{19}	0
170	$-2(x-1)(x^4+2x^2-1)$	-2^{30}	2^{19}	1
171	$-2(x^2-2)(x^4-2x^2-1)$	-2^{31}	2^{13}	0
172	$2(x^2+2)(x^4+2x^2-1)$	2^{31}	2^{13}	0
173	$-2(x-1)(x+1)(x^4-2)$	-2^{31}	2^{15}	1
174	$-2(x^2+1)(x^4+4x^2+2)$	-2^{31}	2^{15}	0
175	$2(x-1)(x+1)(x^4-4x^2+2)$	2^{31}	2^{15}	0
176	$2(x^2+1)(x^4-2)$	2^{31}	2^{15}	0
177	$-2x(x^4+2)$	2^{31}	2^{20}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

178	$2x(x^4-2)$	-2^{31}	2^{20}	1
179	$-2x(x^4-4x^2+2)$	2^{31}	2^{20}	0
180	$-2x(x^4+4x^2+2)$	2^{31}	2^{20}	2
181	$2x(2x^4+8x^3+8x^2-1)$	2^{31}	2^{20}	1
182	$-2(x+1)(x^4-4x^3+2x^2+4x-1)$	2^{31}	2^{20}	1
183	$-2(x+1)(2x^4-1)$	-2^{31}	2^{20}	1
184	$-2x(2x^4+8x^3+8x^2-1)$	2^{31}	2^{20}	0
185	$2(x+1)(x^4-4x^3+2x^2+4x-1)$	2^{31}	2^{20}	1
186	$2(x+1)(2x^4-1)$	-2^{31}	2^{20}	2
187	$-2(x+1)(x^4-4x^3-2x^2+4x-1)$	2^{31}	2^{20}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

188	$2(x+1)(x^4-4x^3-2x^2+4x-1)$	2^{31}	2^{20}	1
189	$-2(x^2+1)(x^4+1)$	-2^{32}	2^{14}	0
190	$-2x(x^2-2x-1)(x^2+2x-1)$	2^{32}	2^{14}	0
191	$2(x^2+1)(x^4+1)$	-2^{32}	2^{14}	0
192	$-2(x^2-2)(2x^4-4x^2+1)$	2^{32}	2^{14}	1
193	$-2(x^2+2)(2x^4+4x^2+1)$	-2^{32}	2^{14}	0
194	$-x(x^4-8x^3+12x^2-16x+4)$	-2^{32}	2^{15}	0
195	$x(x^4-8x^3+12x^2-16x+4)$	-2^{32}	2^{15}	0
196	$-x(x^2-4x+2)(x^2+4x+2)$	2^{32}	2^{16}	0
197	$-x(x^4+12x^2+4)$	2^{32}	2^{16}	0

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198	$x(x^4+8x^3+4x^2-16x+4)$	2^{33}	2^{17}	1
199	$-x(x^4+8x^3+4x^2-16x+4)$	2^{33}	2^{17}	0
200	$-x(x^4-8x^3+28x^2-16x+4)$	2^{33}	2^{17}	0
201	$x(x^4-8x^3+28x^2-16x+4)$	2^{33}	2^{17}	1
202	$(x-1)(x^4-4x^3-14x^2+4x+17)$	2^{33}	2^{17}	1
203	$x(4x^4-20x^2-16x+1)$	2^{33}	2^{17}	0
204	$-(x-1)(x^4-4x^3-14x^2+4x+17)$	2^{33}	2^{17}	0
205	$x(4x^4-20x^2+16x+1)$	2^{33}	2^{17}	1
206	$-2(x^2+2)(x^4+2x^2+2)$	-2^{34}	2^{13}	0
207	$2(x^2-2)(x^4-2x^2+2)$	2^{34}	2^{13}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

208	$-2(x^2-2)(x^4-2x^2+2)$	2^{34}	2^{13}	0
209	$2(x^2+2)(x^4+2x^2+2)$	-2^{34}	2^{13}	0
210	$2(x^2+1)(x^4-2x^2-1)$	2^{34}	2^{16}	1
211	$-2(x^2+1)(x^4-2x^2-1)$	2^{34}	2^{16}	0
212	$2(x-1)(x^4+4x^2-4)$	-2^{34}	2^{16}	0
213	$-2(x-1)(x^4+4x^2-4)$	-2^{34}	2^{16}	1
214	$2x(x^4-8x^3+18x^2+8x+1)$	2^{35}	2^{13}	0
215	$-x(x^4-16x^3+60x^2-32x+4)$	2^{35}	2^{16}	1
216	$x(x^4-16x^3+60x^2-32x+4)$	2^{35}	2^{16}	0
217	$2x(x^4+4x^3+10x^2-4x+1)$	2^{35}	2^{17}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

218	$-2x(x^4+4x^3-6x^2-4x+1)$	2^{35}	2^{17}	0
219	$2(x^2+1)(x^4+4x^3+6x^2+4x+3)$	-2^{35}	2^{17}	0
220	$-2(x^2+1)(x^4+4x^3+6x^2+4x+3)$	-2^{35}	2^{17}	0
221	$-2(x-1)(x+1)(x^4+4x^2-4)$	-2^{36}	2^{13}	0
222	$-2(x^2+1)(x^4-4x^2-4)$	2^{36}	2^{13}	0
223	$2(x-1)(x+1)(x^4+4x^2-4)$	-2^{36}	2^{13}	0
224	$2(x^2+1)(x^4-4x^2-4)$	2^{36}	2^{13}	0
225	$-2(x^2-2)(x^4-2)$	-2^{36}	2^{15}	1
226	$-2(x^2+2)(x^4+4x^2+2)$	-2^{36}	2^{15}	0
227	$-2(x^2-2)(x^4-4x^2+2)$	2^{36}	2^{15}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

228	$-2(x^2+2)(x^4-2)$	2^{36}	2^{15}	0
229	$2(x^2+2)(x^4-2)$	2^{36}	2^{15}	1
230	$2(x^2+2)(x^4+4x^2+2)$	-2^{36}	2^{15}	0
231	$2(x^2-2)(x^4-4x^2+2)$	2^{36}	2^{15}	0
232	$2(x^2-2)(x^4-2)$	-2^{36}	2^{15}	0
233	$(x^2-2x-1)(x^2+1)(x^2+2x-1)$	-2^{36}	2^{16}	2
234	$-(x^2-2x-1)(x^2+1)(x^2+2x-1)$	-2^{36}	2^{16}	0
235	$-(x^2+1)(x^2+2x-1)(x^2+2x+3)$	2^{36}	2^{16}	0
236	$(x^2+1)(x^2+2x-1)(x^2+2x+3)$	2^{36}	2^{16}	1
237	$(x^2-2x+2)(x^2-2)(x^2+2x+2)$	2^{37}	2^{14}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

238	$(x^2-2x+2)(x^2+2)(x^2+2x+2)$	-2^{37}	2^{14}	1
239	$-(x^2-2x+2)(x^2+2)(x^2+2x+2)$	-2^{37}	2^{14}	0
240	$-(x^2-2x+2)(x^2-2)(x^2+2x+2)$	2^{37}	2^{14}	0
241	$-2(2x-3)(x^4+4x^3-6x^2-4x+1)$	2^{37}	2^{14}	0
242	$2(2x-3)(x^4+4x^3-6x^2-4x+1)$	2^{37}	2^{14}	0
243	$-(x^2-2x+3)(x^2+1)(x^2+2x-1)$	2^{38}	2^{16}	0
244	$(x^2-2x+3)(x^2+1)(x^2+2x-1)$	2^{38}	2^{16}	1
245	$-2(x-1)(x^4+8x^3+4x^2-16x+4)$	2^{39}	2^{15}	0
246	$2(x-1)(x^4+8x^3+4x^2-16x+4)$	2^{39}	2^{15}	1
247	$-(x^2-2)(x^4+4x^2-4)$	-2^{39}	2^{16}	2

List of 512 genus 2 curves $C/\mathbb{Q}$

248	$(x^2+2)(x^4-4x^2-4)$	2^{39}	2^{16}	2
249	$(x^2+2)(x^4+4x^2-4)$	2^{39}	2^{16}	1
250	$(x^2-2)(x^4-4x^2-4)$	-2^{39}	2^{16}	1
251	$-(x^2+2)(x^4-4x^2-4)$	2^{39}	2^{16}	1
252	$(x^2-2)(x^4+4x^2-4)$	-2^{39}	2^{16}	1
253	$-(x^2-2)(x^4-4x^2-4)$	-2^{39}	2^{16}	0
254	$-(x^2+2)(x^4+4x^2-4)$	2^{39}	2^{16}	0
255	$(x^2+1)(x^4+6x^2-8x+5)$	-2^{39}	2^{17}	1
256	$-(x^2+1)(x^4+6x^2-8x+5)$	-2^{39}	2^{17}	1
257	$-(x^2-2x-1)(x^4-8x^3+18x^2+8x+1)$	2^{40}	2^{14}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

258	$-(x^2+2x-1)(5x^4+8x^3-6x^2-8x+5)$	2^{40}	2^{14}	0
259	$-(x^2-2x-1)(x^4-4x^3-6x^2+4x+1)$	2^{40}	2^{17}	1
260	$-(x^2-2x-1)(x^4-4x^3+10x^2+4x+1)$	2^{40}	2^{17}	1
261	$(x^2+2x-1)(x^4-4x^3-6x^2+4x+1)$	2^{40}	2^{17}	1
262	$-(x^2-2x-1)(3x^4-4x^3-2x^2+4x+3)$	2^{40}	2^{17}	1
263	$-(x^2+1)(x^4-4x^3+2x^2+4x-7)$	2^{40}	2^{19}	0
264	$(x^2+1)(x^4-4x^3+2x^2+4x-7)$	2^{40}	2^{19}	1
265	$(x^2+1)(x^4-4x^3+10x^2-12x+1)$	2^{40}	2^{19}	2
266	$-(x^2+1)(x^4-4x^3+10x^2-12x+1)$	2^{40}	2^{19}	1
267	$-(x^2+1)(3x^4+4x^3-2x^2-4x+3)$	-2^{41}	2^{20}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

268	$-(x^2+1)(x^4+4x^3+10x^2-4x+1)$	-2^{41}	2^{20}	0*
269	$(x^2+1)(x^4-4x^3-6x^2+4x+1)$	-2^{41}	2^{20}	2
270	$-(x^2+1)(x^4+4x^3-6x^2-4x+1)$	-2^{41}	2^{20}	0
271	$(x^2+1)(x^4-4x^3+10x^2+4x+1)$	-2^{41}	2^{20}	2
272	$(x^2+1)(3x^4-4x^3-2x^2+4x+3)$	-2^{41}	2^{20}	0
273	$-(x^2+1)(x^4+4x^3-6x^2+12x-7)$	2^{42}	2^{15}	0
274	$(x^2+1)(x^4+4x^3-6x^2+12x-7)$	2^{42}	2^{15}	1
275	$-(x^2-2x+3)(x^2+1)(3x^2+2x+1)$	-2^{42}	2^{16}	0
276	$(x^2+1)(x^2+2x+3)(3x^2-2x+1)$	-2^{42}	2^{16}	0
277	$-(x^2-2x-1)(x^2+2x-1)(x^2+2x+3)$	-2^{43}	2^{15}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

278	$(x^2-2x-1)(x^2+2x-1)(x^2+2x+3)$	-2^{43}	2^{15}	1
279	$(x^2+1)(3x^4+4x^3+14x^2+12x+11)$	-2^{43}	2^{17}	1
280	$-(x^2+1)(3x^4-4x^3+14x^2-12x+11)$	-2^{43}	2^{17}	0
281	$(x^2-2)(x^4+4x^3+4x^2-8x+4)$	2^{44}	2^{17}	1
282	$-(x^2-2)(x^4+4x^3+4x^2-8x+4)$	2^{44}	2^{17}	0
283	$-(x^2+2)(x^4-4x^3+12x^2-8x+4)$	-2^{44}	2^{17}	0
284	$(x^2+2x-1)(x^4+6x^2-8x+5)$	2^{44}	2^{17}	1
285	$-(x^2+2x-1)(x^4+6x^2-8x+5)$	2^{44}	2^{17}	0
286	$(x^2+2)(x^4-4x^3+12x^2-8x+4)$	-2^{44}	2^{17}	1
287	$-(x^2+2)(5x^4+4x^3+4x^2+8x+4)$	-2^{44}	2^{17}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

288	$-(x^2+2x+3)(x^4-2x^2-8x+13)$	-2^{44}	2^{17}	0
289	$(x^2+2)(5x^4+4x^3+4x^2+8x+4)$	-2^{44}	2^{17}	0
290	$(x^2+2x+3)(x^4-2x^2-8x+13)$	-2^{44}	2^{17}	1
291	$-(x^2+1)(7x^4+12x^3+30x^2+20x+23)$	-2^{45}	2^{16}	0
292	$(x^2+1)(7x^4-12x^3+30x^2-20x+23)$	-2^{45}	2^{16}	1
293	$(x^2+1)(x^4-8x^3+18x^2+8x+1)$	-2^{45}	2^{18}	2
294	$(x^2+1)(5x^4-8x^3-6x^2+8x+5)$	-2^{45}	2^{18}	0
295	$-(x^2+1)(5x^4+8x^3-6x^2-8x+5)$	-2^{45}	2^{18}	0
296	$-(x^2+1)(x^4+8x^3+18x^2-8x+1)$	-2^{45}	2^{18}	0*
297	$-(x^2+2)(x^4+4x^3+4x^2-8x+4)$	-2^{46}	2^{18}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

298	$(x^2+2)(x^4+4x^3+4x^2-8x+4)$	-2^{46}	2^{18}	2
299	$-(x^2+2x-1)(x^4+6x^2+8x+5)$	2^{46}	2^{18}	0
300	$(x^2-2x-1)(x^4+6x^2-8x+5)$	2^{46}	2^{18}	2
301	$(x^2+2x+3)(x^4+6x^2-8x+5)$	-2^{46}	2^{18}	2
302	$(x^2-2x-1)(x^4+8x^3+22x^2+16x+5)$	2^{46}	2^{18}	2
303	$(x^2+2x+3)(5x^4+8x^3+6x^2+1)$	-2^{46}	2^{18}	1
304	$(x^2+2x-1)(x^4-2x^2-8x+13)$	2^{46}	2^{18}	1
305	$-(x^2+2x+3)(5x^4+8x^3+6x^2+1)$	-2^{46}	2^{18}	1
306	$-(x^2+2x-1)(x^4-2x^2-8x+13)$	2^{46}	2^{18}	1
307	$-(x^2-2x+3)(x^4+6x^2+8x+5)$	-2^{46}	2^{18}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

308	$-(x^2-2x-1)(x^4+8x^3+22x^2+16x+5)$	2^{46}	2^{18}	0
309	$-(x^2-2x-1)(x^2-2x+3)(3x^2+2x+1)$	2^{47}	2^{13}	0
310	$(x^2-4x+2)(x^2-2)(x^2+4x+2)$	2^{47}	2^{14}	1
311	$(x^2+2)(x^4+12x^2+4)$	-2^{47}	2^{14}	1
312	$-(x^2+2)(x^4+12x^2+4)$	-2^{47}	2^{14}	0
313	$-(x^2-4x+2)(x^2-2)(x^2+4x+2)$	2^{47}	2^{14}	0
314	$-(x^2+2x-1)(x^4+4x^3-6x^2+12x-7)$	-2^{47}	2^{16}	0
315	$(x^2+2)(x^4-8x^3+12x^2-16x+4)$	2^{47}	2^{16}	1
316	$(x^2+2x-1)(x^4+4x^3-6x^2+12x-7)$	-2^{47}	2^{16}	1
317	$-(x^2+2)(x^4-8x^3+12x^2-16x+4)$	2^{47}	2^{16}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

318	$(x^2-2)(x^4+8x^3+4x^2-16x+4)$	2^{48}	2^{17}	1
319	$-(x^2-2)(x^4+8x^3+4x^2-16x+4)$	2^{48}	2^{17}	1
320	$(x^2-2x-1)(x^4-4x^3+10x^2+20x+9)$	2^{48}	2^{17}	1
321	$-(x^2+2)(3x^4-8x^3+20x^2-16x+12)$	-2^{48}	2^{17}	0
322	$(x^2+2)(3x^4+8x^3+20x^2+16x+12)$	-2^{48}	2^{17}	0
323	$-(x^2-2x-1)(x^4-4x^3+10x^2+20x+9)$	2^{48}	2^{17}	1
324	$-(x^2-2x-1)(x^4-12x^3+18x^2+44x+17)$	2^{50}	2^{15}	0
325	$(x^2+2)(7x^4-16x^3+36x^2-32x+28)$	-2^{50}	2^{15}	0
326	$(x^2-2x-1)(x^4-12x^3+18x^2+44x+17)$	2^{50}	2^{15}	1
327	$-(x^2+2)(7x^4+16x^3+36x^2+32x+28)$	-2^{50}	2^{15}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

328	$(x^2-2)(x^4+12x^2+4)$	2^{51}	2^{16}	2
329	$-(x^2-4x+2)(x^2+2)(x^2+4x+2)$	-2^{51}	2^{16}	1
330	$(x^2-4x+2)(x^2+2)(x^2+4x+2)$	-2^{51}	2^{16}	1
331	$-(x^2-2)(x^4+12x^2+4)$	2^{51}	2^{16}	0
332	$(x^2-2x-1)(x^2+2x+3)(3x^2-2x+1)$	2^{51}	2^{16}	0
333	$-(x^2+2x-1)(x^4-4x^3-6x^2-12x-7)$	-2^{51}	2^{18}	0
334	$(x^2+2x-1)(x^4-4x^3-6x^2-12x-7)$	-2^{51}	2^{18}	2
335	$-(x^2-2x-1)(x^4+12x^3+34x^2+20x+1)$	2^{52}	2^{19}	2
336	$(x^2+2x-1)(x^4+4x^3-14x^2-4x+17)$	2^{52}	2^{19}	1
337	$-(x^2+2x-1)(x^4+4x^3-14x^2-4x+17)$	2^{52}	2^{19}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

338	$(x^2-2x-1)(x^4+12x^3+34x^2+20x+1)$	2^{52}	2^{19}	1
339	$(x^2-2x+3)(x^4-4x^3-6x^2-12x-7)$	2^{53}	2^{19}	2
340	$(x^2-2x+3)(7x^4-12x^3+6x^2-4x-1)$	2^{53}	2^{19}	1
341	$-(x^2-2x+3)(7x^4-12x^3+6x^2-4x-1)$	2^{53}	2^{19}	0
342	$-(x^2+2x+3)(x^4+4x^3-6x^2+12x-7)$	2^{53}	2^{19}	1
343	$-(x^2-2)(3x^4+8x^3-12x^2-16x+44)$	2^{54}	2^{17}	1
344	$(x^2-2)(11x^4+8x^3-12x^2-16x+12)$	2^{54}	2^{17}	0
345	$(x^2-2)(3x^4+8x^3-12x^2-16x+44)$	2^{54}	2^{17}	0
346	$-(x^2-2)(11x^4+8x^3-12x^2-16x+12)$	2^{54}	2^{17}	1
347	$(x^2+2)(x^4-8x^3+4x^2+16x+4)$	-2^{54}	2^{20}	2

List of 512 genus 2 curves $C/\mathbb{Q}$

348	$-(x^2+2)(x^4+8x^3+4x^2-16x+4)$	-2^{54}	2^{20}	0
349	$-(x^2-2x-1)(x^4+4x^3+10x^2-20x+9)$	2^{54}	2^{20}	1
350	$-(x^2+2x-1)(3x^4+4x^3+14x^2+12x+11)$	2^{54}	2^{20}	1
351	$(x^2-2x-1)(x^4+4x^3+10x^2-20x+9)$	2^{54}	2^{20}	1
352	$(x^2+2x-1)(3x^4+4x^3+14x^2+12x+11)$	2^{54}	2^{20}	1
353	$-(3x^2+4x+2)(x^4+12x^2+4)$	-2^{57}	2^{17}	1
354	$-(x^2+2x+3)(x^4-4x^3+18x^2-28x+17)$	-2^{57}	2^{17}	0
355	$(3x^2+4x+2)(x^4+12x^2+4)$	-2^{57}	2^{17}	0
356	$(x^2+2x+3)(x^4-4x^3+18x^2-28x+17)$	-2^{57}	2^{17}	1
357	$(x^2+2)(3x^4+16x^3+12x^2-32x+12)$	-2^{60}	2^{20}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

358	$-(x^2+2)(3x^4+16x^3+12x^2-32x+12)$	-2^{60}	2^{20}	0
359	$(x^2-2x-1)(x^4+12x^3+18x^2-44x+17)$	2^{60}	2^{20}	1
360	$(x^2+2x-1)(7x^4+12x^3+30x^2+20x+23)$	2^{60}	2^{20}	1
361	$-(x^2-2x-1)(x^4+12x^3+18x^2-44x+17)$	2^{60}	2^{20}	1
362	$-(x^2+2x-1)(7x^4+12x^3+30x^2+20x+23)$	2^{60}	2^{20}	1
363	$-(2x^2-1)(4x^4-36x^2+32x+17)$	2^{60}	2^{20}	0
364	$(x^2-2x-1)(x^4-4x^3-30x^2+4x+97)$	2^{60}	2^{20}	1
365	$(2x^2-1)(4x^4-36x^2+32x+17)$	2^{60}	2^{20}	1
366	$-(x^2-2x-1)(x^4-4x^3-30x^2+4x+97)$	2^{60}	2^{20}	0
367	$-(2x-1)(x^2-2x+3)(x^2+2)$	$2^{16}3^{12}$	2^{10}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

368	$x(x+4)(2x-1)(x^2+2)$	$-2^{21}3^{12}$	2^{11}	0
369	$-x(x+4)(2x-1)(x^2+2)$	$-2^{21}3^{12}$	2^{11}	0
370	$(x-4)x(x^4+8x^3-8x^2+8)$	$-2^{21}3^{12}$	2^{15}	2
371	$-(2x+1)(x^4-4x^2-8x+2)$	$-2^{21}3^{12}$	2^{15}	1
372	$(2x+1)(x^4-4x^2-8x+2)$	$-2^{21}3^{12}$	2^{15}	0
373	$-(x-2)(2x^4+8x^3-4x^2+1)$	$-2^{21}3^{12}$	2^{20}	0
374	$(x-2)(2x^4+8x^3-4x^2+1)$	$-2^{21}3^{12}$	2^{20}	1
375	$-(x^2+2)(x^4-4x^3+2x^2-4x+7)$	$2^{22}3^{12}$	2^{13}	1
376	$(x^2+2)(x^4-4x^3+2x^2-4x+7)$	$2^{22}3^{12}$	2^{13}	1
377	$-(3x^2+4x+4)(x^4-8x^3-8x^2+8)$	$2^{22}3^{12}$	2^{14}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

378	$-x(x^4-14x^2+81)$	$2^{26}3^{12}$	2^{12}	0
379	$-2(2x-1)(x^2-2x+3)(x^2+2)$	$2^{26}3^{12}$	2^{12}	0
380	$(x^2+2)(x^4-4x^2-8x+2)$	$2^{26}3^{12}$	2^{15}	2
381	$-(x^2+2)(x^4-4x^2-8x+2)$	$2^{26}3^{12}$	2^{15}	0
382	$-(2x+1)(x^4+8x^3-8x^2+8)$	$-2^{27}3^{12}$	2^{13}	0
383	$-(2x-1)(x^4-8x^3-8x^2+8)$	$-2^{27}3^{12}$	2^{13}	0
384	$-x(x+4)(x^4-4x^2+8x+2)$	$-2^{27}3^{12}$	2^{14}	1
385	$(x-4)x(x^4-4x^2-8x+2)$	$-2^{27}3^{12}$	2^{14}	1
386	$-(x-1)(x^4+40x^3+20x^2+16x+4)$	$-2^{31}3^{12}$	2^{14}	1
387	$(x-1)(x^4+40x^3+20x^2+16x+4)$	$-2^{31}3^{12}$	2^{14}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

388	$-2(2x+1)(x^4-4x^2-8x+2)$	$-2^{31}3^{12}$	2^{15}	0
389	$-2(x-2)(2x^4+8x^3-4x^2+1)$	$-2^{31}3^{12}$	2^{20}	2
390	$2(x-2)(2x^4+8x^3-4x^2+1)$	$-2^{31}3^{12}$	2^{20}	1
391	$-3(x^2-2)(x^2+1)(2x^2-1)$	$-2^{16}3^{22}$	2^{10}	0
392	$3(x^2-2)(x^2+1)(2x^2-1)$	$-2^{16}3^{22}$	2^{10}	0
393	$2(x^2+2)(x^4-4x^3+2x^2-4x+7)$	$2^{32}3^{12}$	2^{14}	0
394	$-(x+1)(4x^4-16x^3+20x^2-40x+1)$	$-2^{35}3^{12}$	2^{16}	0
395	$(x+1)(4x^4-16x^3+20x^2-40x+1)$	$-2^{35}3^{12}$	2^{16}	1
396	$-2x(x^4-14x^2+81)$	$2^{36}3^{12}$	2^{10}	0
397	$-2(x^2+2)(x^4-4x^2-8x+2)$	$2^{36}3^{12}$	2^{15}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

398	$2(x^2+2)(x^4-4x^2-8x+2)$	$2^{36}3^{12}$	2^{15}	0
399	$-x(2x^2-8x+9)(2x^2+8x+9)$	$2^{36}3^{12}$	2^{16}	1
400	$-x(4x^4+28x^2+81)$	$2^{36}3^{12}$	2^{16}	1
401	$3(x^2-2)(x^2+1)(x^2+4)$	$2^{21}3^{22}$	2^{11}	0
402	$-3(x^2-2)(x^2+1)(x^2+4)$	$2^{21}3^{22}$	2^{11}	0
403	$-3(x^2+4)(x^4-16x^2-8)$	$2^{21}3^{22}$	2^{15}	1
404	$-3(x^2+1)(x^4+8x^2-2)$	$2^{21}3^{22}$	2^{15}	0
405	$3(x^2+1)(x^4+8x^2-2)$	$2^{21}3^{22}$	2^{15}	1
406	$-3(x^2-2)(2x^4-8x^2-1)$	$-2^{22}3^{22}$	2^{13}	1
407	$3(x^2-2)(2x^4-8x^2-1)$	$-2^{22}3^{22}$	2^{13}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

408	$3(x^2-2x-1)(x^2-2x+2)(x^2+4x+2)$	$-2^{22}3^{22}$	2^{13}	0
409	$-3(x^2-4x+2)(x^2+2x-1)(x^2+2x+2)$	$-2^{22}3^{22}$	2^{13}	0
410	$-3(x^2-8)(x^4-16x^2-8)$	$-2^{22}3^{22}$	2^{14}	0
411	$2(x-1)(x^4+40x^3+20x^2+16x+4)$	$-2^{41}3^{12}$	2^{14}	1
412	$-2(x-1)(x^4+40x^3+20x^2+16x+4)$	$-2^{41}3^{12}$	2^{14}	0
413	$-6(x^2-2)(x^2+1)(2x^2-1)$	$-2^{26}3^{22}$	2^{12}	0
414	$6(x^2-2)(x^2+1)(2x^2-1)$	$-2^{26}3^{22}$	2^{12}	0
415	$-3(x^2-2)(x^4+8x^2-2)$	$-2^{26}3^{22}$	2^{15}	1
416	$3(x^2-2)(x^4+8x^2-2)$	$-2^{26}3^{22}$	2^{15}	1
417	$3(x^2+1)(x^4-16x^2-8)$	$2^{27}3^{22}$	2^{13}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

418	$-3(x^2+1)(x^4-16x^2-8)$	$2^{27}3^{22}$	2^{13}	0
419	$3(x^2+4)(x^4+8x^2-2)$	$2^{27}3^{22}$	2^{14}	1
420	$-3(x^2+4)(x^4+8x^2-2)$	$2^{27}3^{22}$	2^{14}	1
421	$-6(x^2+1)(x^4+8x^2-2)$	$2^{31}3^{22}$	2^{15}	1
422	$6(x^2-2)(2x^4-8x^2-1)$	$-2^{32}3^{22}$	2^{14}	0
423	$6(x^2-2x-1)(x^2-2x+2)(x^2+4x+2)$	$-2^{32}3^{22}$	2^{14}	0
424	$-6(x^2-4x+2)(x^2+2x-1)(x^2+2x+2)$	$-2^{32}3^{22}$	2^{14}	0
425	$-(2x^2+1)(4x^4-4x^2+32x-31)$	$2^{50}3^{12}$	2^{15}	0
426	$(2x^2+1)(4x^4-4x^2-32x-31)$	$2^{50}3^{12}$	2^{15}	1
427	$-(x^2-2)(x^2+2)(7x^2-16x-14)$	$-2^{51}3^{12}$	2^{11}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

428	$(x^2-2)(x^2+2)(7x^2+16x-14)$	$-2^{51}3^{12}$	2^{11}	0
429	$6(x^2-2)(x^4+8x^2-2)$	$-2^{36}3^{22}$	2^{15}	0
430	$-6(x^2-2)(x^4+8x^2-2)$	$-2^{36}3^{22}$	2^{15}	1
431	$-3(x^2-2x-1)(x^2+4x+5)(5x^2-4x+1)$	$2^{41}3^{22}$	2^{14}	0
432	$-3(x^2+2x+2)(x^4+16x^3-4x^2+32x+4)$	$2^{45}3^{22}$	2^{16}	1
433	$3(x^2-2x+2)(x^4-16x^3-4x^2-32x+4)$	$2^{45}3^{22}$	2^{16}	0
434	$-3(x^2-6x+7)(x^2+1)(7x^2+6x+1)$	$-2^{46}3^{22}$	2^{16}	0
435	$3(x^2+1)(x^2+6x+7)(7x^2-6x+1)$	$-2^{46}3^{22}$	2^{16}	0
436	$3(x^2+4x+2)(x^4-16x^3-4x^2-32x+4)$	$-2^{50}3^{22}$	2^{15}	0
437	$-3(x^2-4x+2)(x^4+16x^3-4x^2+32x+4)$	$-2^{50}3^{22}$	2^{15}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

438	$3(x^2-2)(x^4+68x^2+4)$	$2^{51}3^{22}$	2^{11}	0
439	$-3(x^2-2)(x^4+68x^2+4)$	$2^{51}3^{22}$	2^{11}	0
440	$3(x^2-2x-1)(17x^4+4x^3+34x^2-4x+17)$	$2^{51}3^{22}$	2^{14}	0
441	$3(x^2-6x+7)(x^4-12x^3-46x^2-84x-47)$	$-2^{60}3^{22}$	2^{20}	0
442	$-3(2x^2+4x+1)(4x^4+16x^3-76x^2+104x-47)$	$-2^{60}3^{22}$	2^{20}	1
443	$-3(x^2-6x+7)(x^4-12x^3-46x^2-84x-47)$	$-2^{60}3^{22}$	2^{20}	1
444	$3(2x^2+4x+1)(4x^4+16x^3-76x^2+104x-47)$	$-2^{60}3^{22}$	2^{20}	0
445	$(x^2+4)(x^4+8x^3+4x^2-16x+28)$	$2^{16}5^{12}$	2^{12}	1
446	$(4x-1)(4x^4-20x^2+16x+7)$	$-2^{16}5^{12}$	2^{12}	0
447	$-(2x+1)(x^4+4x^3-14x^2-4x+41)$	$-2^{26}5^{12}$	2^{12}	1

List of 512 genus 2 curves $C/\mathbb{Q}$

448	$-(x^2+1)(4x^4-16x^3+4x^2+8x+7)$	$2^{26}5^{12}$	2^{12}	0
449	$-2(2x-1)(x^4-4x^3-14x^2+4x+41)$	$-2^{36}5^{12}$	2^{13}	0
450	$-2(x^2+1)(4x^4-16x^3+4x^2+8x+7)$	$2^{36}5^{12}$	2^{13}	0
451	$2(2x-1)(x^4-4x^3-14x^2+4x+41)$	$-2^{36}5^{12}$	2^{13}	0
452	$2(x^2+1)(4x^4-16x^3+4x^2+8x+7)$	$2^{36}5^{12}$	2^{13}	0
453	$(x-3)(4x^4+16x^3-12x^2+8x-47)$	$-2^{36}5^{12}$	2^{16}	0
454	$-(x-3)(4x^4+16x^3-12x^2+8x-47)$	$-2^{36}5^{12}$	2^{16}	0
455	$(2x^2-2x+1)(4x^4+32x^3+76x^2+32x-41)$	$2^{40}5^{12}$	2^{19}	1
456	$-(x^2+2x+2)(23x^4-24x^3-52x^2+80x-28)$	$2^{40}5^{12}$	2^{19}	1
457	$(x^2-2x+2)(23x^4+24x^3-52x^2-80x-28)$	$2^{40}5^{12}$	2^{19}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

458	$-(2x^2-2x+1)(4x^4+32x^3+76x^2+32x-41)$	$2^{40}5^{12}$	2^{19}	0
459	$-5(x^2+2)(x^4+14x^2-1)$	$2^{21}5^{22}$	2^{12}	0
460	$5(x^2-2)(x^4-14x^2-1)$	$-2^{21}5^{22}$	2^{12}	0
461	$-5(x^2-2)(x^4-14x^2-1)$	$-2^{21}5^{22}$	2^{12}	0
462	$5(x^2+2)(x^4+14x^2-1)$	$2^{21}5^{22}$	2^{12}	0
463	$5(3x^2+2x+3)(7x^4-4x^3-14x^2-4x+7)$	$2^{21}5^{22}$	2^{13}	1
464	$-5(x^2-6x+1)(7x^4-4x^3-14x^2-4x+7)$	$-2^{21}5^{22}$	2^{13}	1
465	$(2x^2-2x+1)(4x^4-16x^3-12x^2-8x-47)$	$2^{46}5^{12}$	2^{16}	1
466	$-(2x^2-2x+1)(4x^4-16x^3-12x^2-8x-47)$	$2^{46}5^{12}$	2^{16}	1
467	$-10(x^2+2)(x^4+14x^2-1)$	$2^{31}5^{22}$	2^{13}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

468	$10(x^2-2)(x^4-14x^2-1)$	$-2^{31}5^{22}$	2^{13}	0
469	$5(3x^2+2x+1)(x^4+28x^3-30x^2+36x-31)$	$2^{51}5^{22}$	2^{12}	0
470	$-5(3x^2-2x+1)(x^4-28x^3-30x^2-36x-31)$	$2^{51}5^{22}$	2^{12}	0
471	$-5(x^2-4x+2)(x^4+32x^3+60x^2+64x+4)$	$-2^{51}5^{22}$	2^{14}	0
472	$5(x^2-4x+2)(x^4+32x^3+60x^2+64x+4)$	$-2^{51}5^{22}$	2^{14}	0
473	$-(x^2-2x-1)(2x^4+8x^3+8x^2-8x+7)$	$2^{22}7^{12}$	2^{13}	1
474	$(x^2-2x-1)(2x^4+8x^3+8x^2-8x+7)$	$2^{22}7^{12}$	2^{13}	0
475	$(x^2-4x-4)(x^4+8x^3+16x^2-32x+56)$	$2^{22}7^{12}$	2^{14}	1
476	$-(2x-1)(x^4-8x^2+32x+136)$	$2^{27}7^{12}$	2^{13}	1
477	$(2x-1)(x^4-8x^2+32x+136)$	$2^{27}7^{12}$	2^{13}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

478	$-(4x+1)(2x^4-4x^2-8x+17)$	$2^{27}7^{12}$	2^{14}	1
479	$-(4x-1)(2x^4-4x^2+8x+17)$	$2^{27}7^{12}$	2^{14}	0
480	$-2(x^2+2x-1)(2x^4-8x^3+8x^2+8x+7)$	$2^{32}7^{12}$	2^{14}	0
481	$(x-3)(4x^4+16x^3+84x^2+200x+289)$	$2^{37}7^{12}$	2^{16}	0
482	$(x+3)(4x^4-16x^3+84x^2-200x+289)$	$2^{37}7^{12}$	2^{16}	0
483	$-7(x^2+2)(2x^4-20x^2+1)$	$-2^{22}7^{22}$	2^{13}	1
484	$7(x^2+2)(2x^4-20x^2+1)$	$-2^{22}7^{22}$	2^{13}	0
485	$-7(x^2+8)(x^4-40x^2+8)$	$-2^{22}7^{22}$	2^{14}	0
486	$(x^2-2x-1)(x^4+4x^3+66x^2-4x+577)$	$2^{52}7^{12}$	2^{14}	1
487	$-(x^2-2x-1)(x^4+4x^3+66x^2-4x+577)$	$2^{52}7^{12}$	2^{14}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

488	$7(x^2+1)(x^4-40x^2+8)$	$-2^{27}7^{22}$	2^{13}	1
489	$-7(x^2+1)(x^4-40x^2+8)$	$-2^{27}7^{22}$	2^{13}	0
490	$-7(x^2+4)(x^4-20x^2+2)$	$-2^{27}7^{22}$	2^{14}	1
491	$7(x^2+4)(x^4-20x^2+2)$	$-2^{27}7^{22}$	2^{14}	0
492	$14(x^2+2)(2x^4-20x^2+1)$	$-2^{32}7^{22}$	2^{14}	1
493	$-7(x^2+2x+2)(x^4+32x^3-132x^2+64x+4)$	$-2^{47}7^{22}$	2^{16}	0
494	$7(x^2-2x+2)(x^4-32x^3-132x^2-64x+4)$	$-2^{47}7^{22}$	2^{16}	0
495	$-7(x^2+2x+3)(31x^4-100x^3+30x^2+36x-1)$	$-2^{52}7^{22}$	2^{13}	0
496	$7(x^2+2x+3)(31x^4-100x^3+30x^2+36x-1)$	$-2^{52}7^{22}$	2^{13}	0
497	$-(x-1)(x+1)(x^2+1)(239x^2+2x-239)$	$-2^{9}13^{12}$	2^{8}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

498	$(5x+12)(12x-5)(x^2+1)(x^2+2x-1)$	$-2^{19}13^{12}$	2^{12}	0
499	$x(x^4-478x^2+57122)$	$2^{19}13^{12}$	2^{18}	1
500	$x(x^4+478x^2+57122)$	$2^{19}13^{12}$	2^{18}	1
501	$-2(5x+12)(12x-5)(x^2+1)(x^2+2x-1)$	$-2^{29}13^{12}$	2^{14}	0
502	$2(5x+12)(12x-5)(x^2+1)(x^2+2x-1)$	$-2^{29}13^{12}$	2^{14}	0
503	$2x(x^4+478x^2+57122)$	$2^{29}13^{12}$	2^{18}	1
504	$2x(x^4-478x^2+57122)$	$2^{29}13^{12}$	2^{18}	1
505	$21(x^2+4x+8)(17x^4-32x^3-44x^2+80x-4)$	$-2^{17}3^{22}7^{22}$	2^{13}	0
506	$21(2x^2-2x+1)(x^4+40x^3+44x^2-64x-68)$	$-2^{27}3^{22}7^{22}$	2^{13}	0
507	$42(2x^2-2x+1)(x^4+40x^3+44x^2-64x-68)$	$-2^{37}3^{22}7^{22}$	2^{14}	0

List of 512 genus 2 curves $C/\mathbb{Q}$

508	$-42(2x^2-2x+1)(x^4+40x^3+44x^2-64x-68)$	$-2^{37}3^{22}7^{22}$	2^{14}	0
509	$(x+44)(x^4-16x^3-164x^2+1056x-3388)$	$-2^{39}3^{12}11^{12}$	2^{16}	1
510	$-(x+44)(x^4-16x^3-164x^2+1056x-3388)$	$-2^{39}3^{12}11^{12}$	2^{16}	1
511	$(3x^2+2x+1)(x^4-4x^3-254x^2-252x-2047)$	$2^{54}3^{12}11^{12}$	2^{11}	0
512	$-(3x^2+2x+1)(x^4-4x^3-254x^2-252x-2047)$	$2^{54}3^{12}11^{12}$	2^{11}	0

Summary

Theorem (V. WIP)

There are at least 512 $\mathbb{Q}$-isomorphism classes of genus 2 curves $C/\mathbb{Q}$ whose Jacobian has good reduction away from 2. These include all such curves where $C/\mathbb{Q}$ has good reduction away from either $\{2,3\}$, $\{2,5\}$, or $\{2,7\}$. In particular,

1. *There are exactly 78 genus 2 curves $C/\mathbb{Q}$ whose Jacobian has good reduction away from 2 and such that $rad(\Delta_{min}) = 6$.*
2. *There are exactly 28 genus 2 curves $C/\mathbb{Q}$ whose Jacobian has good reduction away from 2 and such that $rad(\Delta_{min}) = 10$.*
3. *There are exactly 24 genus 2 curves $C/\mathbb{Q}$ whose Jacobian has good reduction away from 2 and such that $rad(\Delta_{min}) = 14$.*

All genus 2 curves $C/\mathbb{Q}$ whose Jacobian is good outside 2 and such that $|\Delta_{min}| \leq 10^{14}$ is contained in our table.

Summary

- All curves (and more stats) given at: bit.ly/genus2

[1] Terms and conditions apply!

Summary

- All curves (and more stats) given at: `bit.ly/genus2`
- Are there any genus 2 curves $C/\mathbb{Q}$ with $\mathrm{Jac}(C)$ good outside 2 not in our list?

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Summary

- All curves (and more stats) given at: `bit.ly/genus2`
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- If you can find any more curves, you'll win 10 000 Kč.[1]

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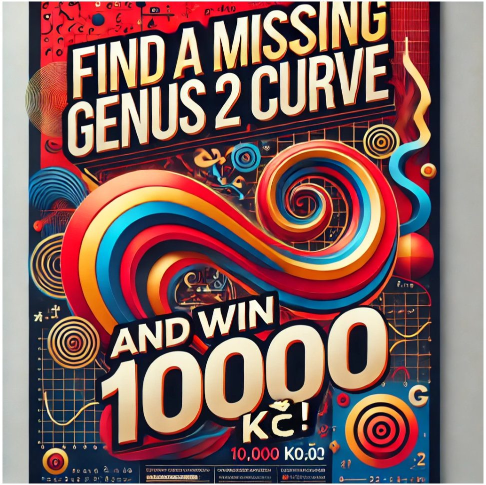
bit.ly/genus2



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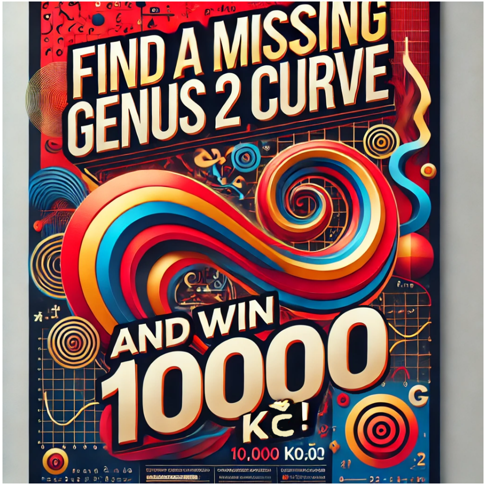

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When the number of known genus 2 curves $C/\mathbb{Q}$ whose Jacobians have good reduction away from 2 is exactly a power of 2